Factorization and the Soft Overlap Contribution to Heavy-to-Light Form Factors

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Abstract

Using the formalism of soft-collinear effective theory, a complete separation of short- and long-distance contributions to heavy-to-light transition form factors at large recoil is performed. The universal functions $\zeta_M(E)$ parameterizing the "soft overlap" contributions to the form factors are defined in terms of matrix elements in the effective theory. Endpoint configurations corresponding to kinematic situations where one of the valence partons in the external mesons carries very small momentum are accounted for in terms of operators involving soft-collinear messenger fields. They contribute at leading order in $\Lambda_{\rm QCD}/E$ and spoil factorization. An analysis of operator mixing and renormalization-group evolution in the effective theory reveals that the intermediate scale $\sqrt{E\Lambda}$ is without significance to the soft functions $\zeta_M(E)$, and that the soft overlap contribution does not receive a significant perturbative (Sudakov) suppression.

1 Introduction

B-meson form factors describing transitions into light pseudoscalar or vector mesons play an essential role in the phenomenology of exclusive weak decays. They enter in semileptonic decays such as $B \to \pi \, l \, \nu$, which provide access to the CKM matrix element $|V_{ub}|$, and in rare decays such as $B \to K^* \gamma$ and $B \to K \, l^+ l^-$, which are sensitive probes for physics beyond the Standard Model. Form factors also enter in the analysis of exclusive hadronic decays such as $B \to \pi\pi$, which give access to the angles γ and α of the unitarity triangle.

In many of these applications form factors are needed near zero momentum transfer ($q^2 \approx 0$), corresponding to a kinematic situation in which a flavor-changing weak current turns a heavy B meson at rest into a highly energetic light meson. Such heavy-to-light transitions at high recoil are suppressed in QCD by inverse powers of the heavy-quark mass $m_b \gg \Lambda_{\rm QCD}$. (This is contrary to the case of heavy-to-heavy transitions such as $B \to D$, which are unsuppressed in the heavy-quark limit.) The challenge in understanding the physics of these processes is to describe properly the transformation of the soft constituents of the B meson into the fast moving, collinear constituents of the energetic light meson in the final state. At lowest order in perturbation theory this transformation can be achieved by the exchange of a gluon, as depicted in Figure 1. Naive power counting suggests that the virtualities of the gluon propagators in these graphs are parametrically large, of order $E\Lambda$ (with $\Lambda \sim \Lambda_{\rm QCD}$), where

$$E = \frac{m_B^2 + m_M^2 - q^2}{2m_B} \gg \Lambda \tag{1}$$

is the energy of the final-state meson M measured in the B-meson rest frame. We assume that the partons inside the light meson carry a significant fraction of its total energy, and that the light constituents of the B meson carry soft momenta of order Λ. It would then appear that the form factor is governed by a hard gluon exchange, which can be dealt with using perturbative methods for hard exclusive QCD processes [1, 2].

However, the situation is indeed far more complicated. In order to analyse the graphs in Figure 1, it is convenient to work in the B-meson rest frame and choose the outgoing light-meson momentum in the z-direction. We then define two light-like vectors $n^\mu = (1, 0, 0, 1)$ and $\bar{n}^\mu = (1, 0, 0, -1)$ satisfying $n \cdot \bar{n} = 2$. The meson momentum can be written as $p_M^\mu = E n^\mu + O(m_M^2/4E)$, which is nearly light-like. Next, we assign incoming momentum l to the light spectator anti-quark inside the B meson, and we write the outgoing momenta p_1 and p_2 of the final-state quark and anti-quark that are absorbed by the light meson as

$$p_1^\mu = x_1 E n^\mu + p_\perp^\mu + \dots, \qquad p_2^\mu = x_2 E n^\mu - p_\perp^\mu + \dots, \tag{2}$$

where the longitudinal momentum fractions satisfy $x_1 + x_2 = 1$, the transverse momenta are $O(\Lambda)$, and the dots represent terms in the direction of $\bar{n}$ that are $O(\Lambda^2/E)$. Assuming that $x_i = O(1)$ and keeping only terms of leading order in Λ/E, a straightforward calculation of the two diagrams yields the partonic amplitude

$$A = -g^2 \left(\Gamma \frac{m_b(1 + \slashed{v}) - x_2 E \slashed{n}}{4 m_b E^2 x_2^2 \, n \cdot l} \gamma_\mu + \gamma_\mu \frac{E \slashed{n} - \slashed{l}}{4 E^2 x_2 (n \cdot l)^2} \Gamma \right) t_a * \gamma^\mu t_a + \dots, \tag{3}$$
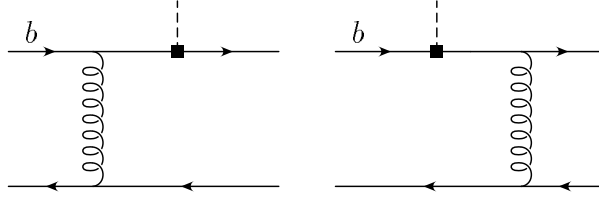
Figure 1: Gluon exchange contributions to heavy-to-light form factors. The flavor-changing current is denoted by a dashed line. The lines to the left belong to the B meson, and those to the right belong to the light meson M.

where $v^\mu = (1,0,0,0)$ is the B-meson velocity, Γ denotes the Dirac structure of the flavor-changing current, the $*$ product means that the two factors must be sandwiched between quark spinors, and the dots represent power-suppressed terms. After projection onto the external meson states, the partonic amplitude must be convoluted with light-cone distribution amplitudes (LCDAs) governing the distributions of x_2 and $n \cdot l$ inside the meson states. For $x_2 = O(1)$, the terms in the parenthesis scale like $1/(E^2\Lambda)$, which after projection onto the meson states implies a power suppression of the corresponding heavy-to-light form factors. (The contribution in the second term that superficially scales like $1/(E\Lambda^2)$ vanishes by the equations of motion.) However, the singularities of the amplitude arising for $x_2 \to 0$ or $n \cdot l \to 0$ are such that the convolution integrals with the LCDAs diverge at the endpoints [3, 4]. Near the endpoints the perturbative analysis of the diagrams in Figure 1 breaks down, since the gluon propagators are no longer far off-shell. This observation is the basis for the notion of a soft overlap contribution to the form factor (Feynman mechanism), which would formally be the leading contribution, since it is not suppressed by a perturbative coupling constant α_s. The precise nature of this contribution and its scaling properties in the heavy-quark limit remain however unclear.

The QCD factorization approach to exclusive hadronic B decays assumes that heavy-to-light form factors are dominated by the soft overlap contribution and are truly non-perturbative objects, which are treated as hadronic input parameters [5]. However, it has sometimes been argued that the summation of large Sudakov logarithms associated with the soft gluon exchange mechanism may lead to a strong suppression of the soft overlap term, essentially reinstating the perturbative nature of the form factors (see, e.g., [6, 7, 8]). The assumption of a short-distance nature of heavy-to-light form factors at large recoil is the basis of the pQCD approach to exclusive hadronic B decays [9], which is often considered a competitor to QCD factorization. Leaving aside some conceptual problems associated with this treatment [10], the issue of Sudakov logarithms is an intricate one, because contributions to the form factors can arise from several different energy scales. Besides the hard scale $E \sim m_B$ and the hadronic scale Λ, interactions between soft and collinear partons involve an intermediate "hard-collinear" scale of order $\sqrt{E\Lambda}$. To settle the question of Sudakov suppression of the soft overlap contribution is one of the main motivations for our work.

Recently, much progress in the understanding of exclusive B decays has been achieved by combining methods developed for hard exclusive processes with concepts of heavy-quark expansions and effective field theory. Soft-collinear effective theory (SCET) has been estab-

lished as a tool to systematically disentangle the various momentum regions and scales relevant in processes involving soft and collinear partons [11, 12, 13, 14, 15, 16]. This approach has already been applied to study some aspects of heavy-to-light form factors at large recoil [15, 16, 17, 18, 19]. Because form factors are power-suppressed quantities, the analysis in the framework of SCET is highly non-trivial. Our main goal in the present work is to provide a rigorous definition of the soft overlap contribution in terms of SCET operators. This will allow us to study systematically its dependence on the large scale E.

Let us start with a summary of related work by previous authors. The discussion of heavy-to-light form factors simplifies in the heavy-quark limit $E \gg \Lambda$ and can be summarized by the factorization formula [20]

$$f_i^{B \to M}(E) = C_i(E, \mu_{\mathrm{I}}) \, \zeta_M(\mu_{\mathrm{I}}, E) + T_i(E, \sqrt{E\Lambda}, \mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) + \dots, \qquad (4)$$

where the dots represent terms that are of subleading order in Λ/E. C_i and T_i are calculable short-distance coefficient functions, ϕ_B and ϕ_M denote the leading-order LCDAs of the B meson and the light meson M, and the $\otimes$ products imply convolutions over light-cone momentum fractions. The functions ζ_M denote universal form factors that only depend on the nature of the light final-state meson but not on the Lorentz structure of the currents whose matrix elements define the various form factors. The first term in the factorization formula therefore implies spin-symmetry relations between different form factors, which were first derived in [21] by considering the large-energy limit of QCD. The second term in (4), which arises from hard-collinear gluon exchange, breaks these symmetry relations [20].

The arguments E and $\sqrt{E\Lambda}$ in the short-distance coefficients C_i and T_i are representative for any of the hard or hard-collinear scales in the problem, respectively. The form of the factorization formula shown in (4) assumes that the factorization scale μ_{I} in the first term is chosen to lie between the hard scale E and the hard-collinear scale $\sqrt{E\Lambda}$, whereas the scale μ in the second term is chosen to lie below the hard-collinear scale. The Wilson coefficients C_i receive contributions at tree level, whereas the hard-scattering kernels T_i start at first order in $\alpha_s(\sqrt{E\Lambda})$. Naively, one would conclude that the spin-symmetry preserving term provides the leading contribution to the form factors in the heavy-quark limit. Only then the notion of an approximate spin symmetry would be justified. However, the situation is more complicated because as written in (4) the universal functions $\zeta_M(\mu_{\mathrm{I}}, E)$ still depend on the short-distance scale $\sqrt{E\Lambda}$; in fact, their E-dependence remains unspecified. Contrary to the hard-scattering term, for which all short-distance physics has been factorized into the kernels T_i, the first term in (4) has so far not been factorized completely. This is reflected in existing discussions of the factorization properties of heavy-to-light form factors in the context of SCET [15, 16, 17, 18]. The matching of a QCD form factor onto matrix elements in SCET is usually done in two steps: $\mathrm{QCD} \to \mathrm{SCET_I} \to \mathrm{SCET_{II}}$. In the first step, hard fluctuations with virtualities on the scale $E \sim m_B$ are integrated out, while in the second step hard-collinear modes with virtualities of order $\sqrt{E\Lambda}$ are removed. Both steps of this matching are understood for the hard-scattering term in the factorization formula, for which the kernels T_i can be factorized as [17]

$$T_i(E, \sqrt{E\Lambda}, \mu) = \sum_j H_{ij}(E, \mu) \otimes J_j(\sqrt{E\Lambda}, \mu). \qquad (5)$$

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On the other hand, the matching onto SCET$_\mathrm{II}$ has not yet been performed for the universal functions $\zeta_M(E, \mu_\mathrm{I})$ entering the first term.[1] In [17, 18] these functions are defined in terms of matrix elements in the intermediate effective theory SCET$_\mathrm{I}$, which leaves open the possibility that they could be dominated by short-distance physics. In the present work we show that the functions ζ_M renormalized at a hard-collinear scale $\mu_{hc} \sim \sqrt{E\Lambda}$ can be factorized further according to

$$\zeta_M(\mu_{hc}, E) = \sum_k D_k^{(M)}(\sqrt{E\Lambda}, \mu_{hc}, \mu) \otimes \xi_k^{(M)}(\mu, E) \,, \tag{6}$$

where the functions $\xi_k^{(M)}$ are defined in terms of hadronic matrix elements of SCET$_\mathrm{II}$ operators. By solving the renormalization-group (RG) equation for these operators we show that the universal form factors ζ_M do not receive a significant perturbative suppression, neither by a power of $\alpha_s(\sqrt{E\Lambda})$ nor by resummed Sudakov logarithms. This statement holds true even in the limit where m_B is taken to be much larger than the physical B-meson mass. Perhaps somewhat surprisingly, we find that the hard-collinear scale $\sqrt{E\Lambda}$ is without physical significance to the soft functions ζ_M. Switching from SCET$_\mathrm{I}$ to SCET$_\mathrm{II}$ one describes the same physics using a different set of degrees of freedom. This observation leads us to the most important conclusion of our paper, namely that the long-distance, soft overlap contribution to heavy-to-light form factors exists. However, we point out that even at low hadronic scales $\mu \sim \Lambda$ the functions $\xi_k^{(M)}$ contain a dependence on the large recoil energy E which is of long-distance nature and cannot be factorized using RG techniques. As a result, it is impossible to determine the asymptotic behavior of the QCD form factors $f_i^{B \to M}$ as $E \to \infty$ using short-distance methods.

Let us comment in more detail on the philosophy adopted by Bauer et al. in [17], where an attempt was made to prove the factorization formula (4) based on hard-collinear/soft factorization of operators in SCET$_\mathrm{I}$. In this interesting paper, the authors introduced the notion of "factorizable operators", in which all soft gluon interactions occur in redefined soft quark fields $\mathcal{H} = Y^\dagger h$ and $\mathcal{Q} = Y^\dagger q$, which in soft light-cone gauge coincide with the original fields for heavy and light quarks. (Here Y is a soft Wilson line in SCET$_\mathrm{I}$.) They argued that the $B \to M$ matrix elements of "factorizable" SCET$_\mathrm{I}$ operators could be written in terms of convergent convolution integrals of hard-scattering kernels with leading-order LCDAs, and that endpoint divergences of convolution integrals only arise in an attempt to evaluate the matrix elements of "non-factorizable" operators. They also showed that the operators which give rise to spin-symmetry breaking contributions to the form factors are all "factorizable". The matrix elements of the "non-factorizable" SCET$_\mathrm{I}$ operators were used to define the universal functions ζ_M. In the present paper we show that hard-collinear/soft factorization in SCET$_\mathrm{I}$ alone is not sufficient to guarantee the absence of endpoint singularities in the convolution integrals that enter the factorization formula. By completing the matching of SCET$_\mathrm{I}$ operators onto the final effective theory SCET$_\mathrm{II}$, we identify a particular "factorizable" SCET$_\mathrm{I}$ operator (called T_0^F in [17]) whose matrix elements cannot be written in the form of convergent convolution integrals of perturbative kernels with LCDAs.

To understand the physics of endpoint singularities it is necessary to perform the matching onto SCET$_\mathrm{II}$ explicitly and study the interplay of the various fields that are contained in this effective theory. For some time it was believed that SCET$_\mathrm{II}$ factorizes into disconnected soft

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[1]The notation μ_I serves as a reminder that the factorization scale for this term is defined in the intermediate effective theory SCET$_\mathrm{I}$.

and collinear sectors. If true, this would imply the absence of endpoint singularities, because they correspond to momentum scalings that are not accounted for by soft or collinear fields. It would also imply that the functions ζ_M are proportional to a power of $\alpha_s(\sqrt{E\Lambda})$ (modulo logarithms), since a hard-collinear gluon exchange is required to transform the soft spectator anti-quark inside the B meson into a collinear anti-quark that can be absorbed by the final-state light meson M. As a result, one would expect that the spin-symmetry relations could receive $O(1)$ corrections that are not suppressed by any small parameter. Recently, however, we have argued that soft/collinear factorization is *not* a generic property of SCET$_{\mathrm{II}}$, not even at leading order in power counting [22, 23]. A non-perturbative cross-talk between the two sectors of the theory exists, which can be dealt with most elegantly by introducing a new kind of fields called soft-collinear messengers.[2] In the present work we use this framework and apply it to the case of heavy-to-light form factors. We show that the endpoint singularities arising in convolution integrals are removed by including the contributions from soft-collinear fields. The sum of all SCET$_{\mathrm{II}}$ matrix elements is free of such singularities. Even at hadronic scales $\mu \sim \Lambda$, the functions ζ_M cannot be written as convolution integrals with LCDAs, meaning that soft and collinear contributions cannot be factorized.

In order to prove the factorization formula (4) one needs to show that all contributions to the form factors that do not obey spin-symmetry relations can be written in terms of convolution integrals involving the leading-order LCDAs of the B meson and the light meson. Specifically, this means showing that (i) no higher Fock states (or higher-twist two-particle distribution amplitudes) contribute at leading power, and (ii) the convolution integrals are convergent to all orders in perturbation theory. Point (i) can be dealt with using the power-counting rules of SCET along with reparameterization invariance [17, 18]. Here we use our formalism to complete step (ii) of the factorization proof. (The convergence of the convolution integral over the B-meson LCDA can also be shown using arguments along the lines of [24, 25].)

The remainder of the paper is structured as follows: After a brief introduction to the SCET$_{\mathrm{II}}$ formalism in Section 2, we discuss in Section 3 the matching of the universal functions ζ_M onto operators in the low-energy effective theory containing only soft and collinear fields. We construct a basis of the operators contributing at leading power and derive the tree-level expressions for their Wilson coefficient functions. We find that the "factorizable" SCET$_{\mathrm{I}}$ operator T_0^{F} (in the language of [17]) matches onto some SCET$_{\mathrm{II}}$ operators whose matrix elements cannot be written as convergent convolutions over B-meson and light-meson LCDAs. These matrix elements are associated with non-valence Fock states of the external mesons, which contribute at the same order in power counting as the two-particle valence Fock states [16, 19]. The appearance of endpoint singularities in convolution integrals signals the relevance of contributions involving long-distance soft-collinear messenger modes. This connection is made more explicit in Section 4, where we analyze in detail a toy model of endpoint singularities. In Section 5 we then complete the parameterization of the universal functions ζ_M in terms of SCET$_{\mathrm{II}}$ matrix elements by including the messenger contributions. The RG mixing of the various SCET$_{\mathrm{II}}$ operators is discussed in Section 6. We show that the linear combination of operators relevant to the form factors is an eigenvector of the RG evolution

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[2]This cross-talk is also seen in the approach of [18], where it appears as a dependence of soft and collinear contributions on a common analytic regulator introduced to define certain one-loop graphs with light-like external momenta that cannot be regularized dimensionally.

matrix and identify the corresponding eigenvalue with the anomalous dimension of a current operator. Solving the evolution equation for this operator, we resum the short-distance Sudakov logarithms arising in the evolution from the hard scale $\mu \sim E$ down to hadronic scales $\mu \sim \Lambda$ at next-to-leading order. Finally, in Section 7 we apply our formalism to prove that the spin-symmetry violating term in the factorization formula (4) is free of endpoint singularities. Our results are summarized in Section 8.

2 Soft-collinear effective theory

The construction of the effective Lagrangian for SCET$_{\text{II}}$ has been discussed in detail in [16, 22, 23], to which we refer the reader for details. Here we summarize the main results and notations needed for our discussion.

In order to account for the fact that different components of particle momenta and fields scale differently with the large scale E, one decomposes 4-vectors in the light-cone basis constructed with the help of the two light-like vectors n^μ and $\bar{n}^\mu$. Specifically, $n^\mu = (1, 0, 0, 1)$ is chosen to be the direction of an outgoing fast hadron (or a jet of hadrons), and $\bar{n}^\mu = (1, 0, 0, -1)$ points in the opposite direction. The light-cone decomposition of a 4-vector reads

$$p^\mu = (n \cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n} \cdot p)\frac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu \,, \tag{7}$$

where $p_\perp \cdot n = p_\perp \cdot \bar{n} = 0$. The relevant SCET$_{\text{II}}$ degrees of freedom describing the partons in the external hadronic states of exclusive B decays are soft or collinear. Introducing an expansion parameter $\lambda = \Lambda/E \ll 1$, we have $p_s^\mu \sim E(\lambda, \lambda, \lambda)$ for soft momenta and $p_c^\mu \sim E(\lambda^2, 1, \lambda)$ for collinear momenta, where we indicate the scaling properties of the components $(n \cdot p, \bar{n} \cdot p, p_\perp)$. The corresponding effective-theory fields and their scaling properties are $h \sim \lambda^{3/2}$ (soft heavy quark), $q_s \sim \lambda^{3/2}$ (soft light quark), $A_s^\mu \sim (\lambda, \lambda, \lambda)$ (soft gluon), and $\xi \sim \lambda$ (collinear quark), $A_c^\mu \sim (\lambda^2, 1, \lambda)$ (collinear gluon). The effective theory also contains soft-collinear quark and gluon messenger fields, $\theta \sim \lambda^2$ and $A_{sc}^\mu \sim (\lambda^2, \lambda, \lambda^{3/2})$, which have couplings to both soft and collinear fields.

The various fields present in the effective theory obey a set of "homogeneous" gauge transformations, which leave their scaling properties unchanged [13, 22, 26]. In these transformations the soft-collinear gluon field is treated as a background field. Soft fields transform under soft and soft-collinear gauge transformations but are invariant under collinear transformations. Likewise, collinear fields transform under collinear and soft-collinear gauge transformations but are invariant under soft transformations. Soft-collinear fields only transform under soft-collinear gauge transformations.

The effective Lagrangian of SCET$_{\text{II}}$ can be split up as

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_s + \mathcal{L}_c + \mathcal{L}_{sc} + \mathcal{L}_{\text{int}}^{(0)} + \dots \,, \tag{8}$$

where the dots represent power-suppressed interactions. None of the terms in the Lagrangian is renormalized beyond the usual renormalization of the fields and coupling constant. The integration measure d^4x in the action $S_{\text{SCET}} = \int d^4x \, \mathcal{L}_{\text{SCET}}$ scales like λ^{-4} for all terms except the soft-collinear Lagrangian $\mathcal{L}_{sc}$, for which it scales like λ^{-6}. The first three terms above

correspond to the Lagrangians of soft particles (including heavy quarks), collinear particles, and soft-collinear particles. They are given by

$$\mathcal{L}_s = \bar{q}_s\, i\slashed{D}_s\, q_s + \bar{h}\, iv \cdot D_s\, h + \mathcal{L}_s^{\text{glue}}\,,$$

$$\mathcal{L}_c = \bar{\xi}\, \frac{\slashed{\bar{n}}}{2}\, in \cdot D_c\, \xi - \bar{\xi}\, i\slashed{D}_{c\perp}\, \frac{\slashed{\bar{n}}}{2}\, \frac{1}{i\bar{n} \cdot D_c}\, i\slashed{D}_{c\perp}\, \xi + \mathcal{L}_c^{\text{glue}}\,, \tag{9}$$

$$\mathcal{L}_{sc} = \bar{\theta}\, \frac{\slashed{\bar{n}}}{2}\, in \cdot D_{sc}\, \theta - \bar{\theta}\, i\slashed{D}_{sc\perp}\, \frac{\slashed{\bar{n}}}{2}\, \frac{1}{i\bar{n} \cdot D_{sc}}\, i\slashed{D}_{sc\perp}\, \theta + \mathcal{L}_{sc}^{\text{glue}}\,,$$

where $iD_s^\mu \equiv i\partial^\mu + gA_s^\mu$ is the covariant derivative built using soft gauge fields, etc. Collinear, soft-collinear, and heavy-quark fields in the effective theory are described by two-component spinors subject to the constraints $\slashed{n}\,\xi = 0$, $\slashed{n}\,\theta = 0$, and $\slashed{v}\,h = h$. The gluon Lagrangians in the three sectors retain the same form as in full QCD, but with the gluon fields restricted to the corresponding subspaces of their soft, collinear, or soft-collinear Fourier modes.

In interactions with other fields, the soft-collinear fields (but not the soft and collinear fields) are multipole expanded about the points x_- (interactions with collinear fields) or x_+ (interactions with soft fields). The leading-order interactions are given by

$$\mathcal{L}_{\text{int}}^{(0)}(x) = \bar{q}_s(x)\, \frac{\slashed{\bar{n}}}{2}\, g\bar{n} \cdot A_{sc}(x_+)\, q_s(x) + \bar{h}(x)\, \frac{n \cdot v}{2}\, g\bar{n} \cdot A_{sc}(x_+)\, h(x)$$

$$+ \bar{\xi}(x)\, \frac{\slashed{\bar{n}}}{2}\, gn \cdot A_{sc}(x_-)\, \xi(x) + \text{pure glue terms}\,. \tag{10}$$

Momentum conservation implies that soft-collinear fields can only couple to either soft or collinear fields, but not both. The gluon self-couplings will not be relevant to our discussion.

At leading order in λ, matching calculations can be done by evaluating diagrams with external quark and gluon states and making the replacements

$$\psi_\xi \to W_c^\dagger\, \xi\,, \qquad \psi_q \to S_s^\dagger\, q_s\,, \qquad \psi_Q \to S_s^\dagger\, h\,,$$

$$gA_{c\perp}^\mu \to W_c^\dagger\, (iD_{c\perp}^\mu W_c)\,, \qquad gA_{s\perp}^\mu \to S_s^\dagger\, (iD_{s\perp}^\mu S_s)\,, \tag{11}$$

where

$$W_c(x) = \mathrm{P}\exp\left(ig \int_{-\infty}^0 dt\, \bar{n} \cdot A_c(x + t\bar{n})\right)\,,$$

$$S_s(x) = \mathrm{P}\exp\left(ig \int_{-\infty}^0 dt\, n \cdot A_s(x + tn)\right) \tag{12}$$

are collinear and soft Wilson lines [13, 15], which effectively put the SCET fields into light-cone gauge. The leading-order interactions between soft-collinear fields and soft or collinear fields in (10) can be removed by a redefinition of the soft and collinear fields, which is analogous to the decoupling of soft gluons in SCET$_\mathrm{I}$ [13]. To this end, one rewrites the fields arising in matching calculations in terms of the gauge-invariant building blocks

$$W_c^\dagger\, \xi = S_{sc}(x_-)\, \mathcal{X}\,, \qquad S_s^\dagger\, q_s = W_{sc}(x_+)\, \mathcal{Q}_s\,, \qquad S_s^\dagger\, h = W_{sc}(x_+)\, \mathcal{H}\,,$$

$$W_c^\dagger\, (iD_{c\perp}^\mu W_c) = S_{sc}(x_-)\, \mathcal{A}_{c\perp}^\mu\, S_{sc}^\dagger(x_-)\,, \qquad S_s^\dagger\, (iD_{s\perp}^\mu S_s) = W_{sc}(x_+)\, \mathcal{A}_{s\perp}^\mu\, W_{sc}^\dagger(x_+)\,, \tag{13}$$

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where fields without argument live at position x. The quantities W_{sc} and S_{sc} are a new set of Wilson lines defined in analogy with W_c and S_s in (12), but with the gluon fields replaced by soft-collinear gluon fields in both cases. The calligraphic fields in (13) are invariant under soft, collinear, and soft-collinear gauge transformations. When they are introduced in the SCET$_\mathrm{II}$ Lagrangian the leading-order interaction terms denoted by $\mathcal{L}_\mathrm{int}^{(0)}$ vanish. Residual interactions between soft-collinear fields and soft or collinear fields start at $O(\lambda^{1/2})$ [22]. We will study some of these interactions in more detail in Section 5.

3 Matching calculations

In the matching of the intermediate effective theory SCET$_\mathrm{I}$ onto the final low-energy effective theory SCET$_\mathrm{II}$, hard-collinear modes with virtuality of order $\sqrt{E\Lambda}$ are integrated out, and their effects are included in short-distance coefficient functions. Our primary goal in this section is to construct a basis of operators relevant to the matching of the universal functions ζ_M in (4) onto the low-energy theory (at leading power in Λ/E), and to calculate their Wilson coefficients at lowest order in perturbation theory. As always, matching calculations can be done using on-shell external quark and gluon states rather than the physical meson states, whose matrix elements define the form factors. The results for the Wilson coefficients are insensitive to infra-red physics. At the end of this section we briefly discuss also the spin-symmetry breaking contributions to the form factors.

3.1 Spin-symmetric contributions

Heavy-to-light form factors are defined in terms of $B \to M$ matrix elements of flavor-changing currents $\bar{q}\,\Gamma\,b$. The spin-symmetric contributions to the form factors are characterized by the fact that in the intermediate effective theory (i.e., after hard fluctuations with virtuality $\mu \sim E \sim m_B$ are integrated out) they contain the Dirac structure Γ sandwiched between the two projection operators $\frac{1}{4}\slashed{n}\slashed{\bar{n}}$ and $\frac{1}{2}(1+\slashed{v})$. This implies that Γ can be decomposed into a linear combination of only three independent Dirac structures, which leads to symmetry relations between various form factors [21]. It has been shown in [17] that in SCET$_\mathrm{I}$ the relevant operators can be written as time-ordered products of the effective Lagrangian with effective current operators $J_M^{(0)}(x) = (\bar{\xi}_{hc}W_{hc})(x)\,\Gamma_M\,h(x_-)$ defined by the matching relation

$$\bar{q}\,\Gamma\,b = \sum_M K_M^\Gamma(m_b, E, \mu)\,J_M^{(0)} + \dots , \qquad (14)$$

where the dots denote power-suppressed terms. Here $\Gamma_M = 1, \gamma_5, \gamma_{\perp\nu}$ is one of the three Dirac basis matrices that remain after the projections onto the two-component spinors $\bar{\xi}_{hc}$ and h. The variable E entering the coefficients K_M^Γ is the total energy carried by collinear particles (more precisely, $2E = \bar{n} \cdot p_c^{\mathrm{tot}}$), which in our case coincides with the energy of the final-state meson M. Since the Lagrangian is a Lorentz scalar, it follows that for $B \to M$ transitions each of the three possibilities corresponds to a particular choice of the final-state meson (hence the label "M" on Γ_M), namely $\Gamma_M = 1$ for $M = P$ a pseudoscalar meson, $\Gamma_M = \gamma_5$ for $M = V_\parallel$ a longitudinally polarized vector meson, and $\Gamma_M = \gamma_{\perp\nu}$ for $M = V_\perp$ a transversely polarized

Table 1: Coefficients K_M^Γ arising in the leading-order matching of flavor-changing currents from QCD onto SCET$_\mathrm{I}$. The coefficients C_i are defined in [12]. We denote $\hat{q} = q/m_B$ and $\hat{E} = E/m_B$, where $q = p_B - p_M$.

Current $\bar{q}\,\Gamma\,b$	$M = P$ $(\Gamma_M = 1)$	$M = V_\parallel$ $(\Gamma_M = \gamma_5)$	$M = V_\perp$ $(\Gamma_M = \gamma_{\perp\nu})$
$\bar{q}\gamma^\mu b$	$(n^\mu C_4 + v^\mu C_5)$	—	$g_\perp^{\mu\nu} C_3$
$\bar{q}\gamma^\mu\gamma_5 b$	—	$-(n^\mu C_7 + v^\mu C_8)$	$-i\epsilon_\perp^{\mu\nu} C_6$
$\bar{q}\,i\sigma^{\mu\nu}\hat{q}_\nu b$	$[v^\mu - (1 - \hat{E})n^\mu]\,C_{11}$	—	$-g_\perp^{\mu\nu}[C_9 + (1 - \hat{E})C_{12}]$
$\bar{q}\,i\sigma^{\mu\nu}\gamma_5\hat{q}_\nu b$	—	$[v^\mu - (1 - \hat{E})n^\mu]\,C_{10}$	$-i\epsilon_\perp^{\mu\nu}(C_9 + \hat{E}C_{12})$

vector meson. The coefficients K_M^Γ for the various currents that are relevant in the discussion of heavy-to-light form factors are summarized in Table 1, where we use the definitions

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu \bar{n}^\nu + n^\nu \bar{n}^\mu}{2}\,, \qquad \epsilon_\perp^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta}\bar{n}_\alpha n_\beta \tag{15}$$

for the symmetric and anti-symmetric tensors in the transverse plane. (We use $\epsilon_{0123} = 1$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.) Once a definition for the heavy-to-light form factors is adopted, it is an easy exercise to read off from the table which of the coefficient functions C_i contribute to a given form factor. This determines the functions $C_i(E, \mu_\mathrm{I})$ in (4). (In general, these functions are linear combinations of the C_i in Table 1.)

Our goal is to match the time-ordered product $i \int d^4x\, \mathrm{T}\,\{J_M^{(0)}(0), \mathcal{L}_{\mathrm{SCET_I}}(x)\}$ onto operators in SCET$_\mathrm{II}$, focusing first on operators which include soft and collinear fields only. The insertion of a subleading interaction from the SCET$_\mathrm{I}$ Lagrangian is required to transform the soft B-meson spectator anti-quark into a hard-collinear final-state parton. In the resulting SCET$_\mathrm{II}$ interaction terms the soft and collinear fields must be multipole expanded, since their momenta have different scaling properties with the expansion parameter λ [15]. Soft fields are expanded about $x_+ = 0$ while collinear ones are expanded about $x_- = 0$. In general, collinear fields can live on different x_+ positions while soft fields can live on different x_- positions. The resulting non-localities of SCET$_\mathrm{II}$ operators are allowed by gauge invariance and are needed to reproduce the dependence of decay amplitudes on the longitudinal momentum components $\bar{n} \cdot p_c$ and $n \cdot p_s$ of collinear and soft momenta. The relevant operators can be written as matrix elements of color singlet-singlet four-quark operators multiplied by position-dependent Wilson coefficients, i.e. [16, 23]

$$\int ds\, dt\, \widetilde{D}_k(s, t, E, \mu)\,[(\bar{\xi}W_c)(x_+ + x_\perp) \ldots (W_c^\dagger \xi)(x_+ + x_\perp + s\bar{n})]$$
$$\times [(\bar{q}_s\, S_s)(x_- + x_\perp + tn) \ldots (S_s^\dagger h)(x_- + x_\perp)]\,, \tag{16}$$

where the dots represent different Dirac structures. There is no need to include color octet-octet operators, since they have vanishing projections onto physical hadron states and do not

mix into the color singlet-singlet operators under renormalization. The Fourier transforms of the coefficient functions defined as

$$D_k(\omega, u_2, E, \mu) = \int ds \, e^{2iEu_2 s} \int dt \, e^{-i\omega t} \, \widetilde{D}_k(s, t, E, \mu) \tag{17}$$

coincide with the momentum-space Wilson coefficient functions, which we will calculate below. (In the matching calculation, ω is identified with the component $n \cdot l$ of the incoming spectator momentum, and u_2 is identified with the longitudinal momentum fraction x_2 carried by the collinear anti-quark in the final-state meson.) Consider now what happens when we rewrite the operators above in terms of the gauge-invariant building blocks defined in (13). Since the collinear fields are evaluated at $x_- = 0$ and the soft fields at $x_+ = 0$, it follows that all factors of the soft-collinear Wilson lines $S_{sc}(0)$ and $W_{sc}(0)$ cancel out. Hence, we can rewrite the operators in the form (setting $x = 0$ for simplicity)

$$\int ds \, dt \, \widetilde{D}_k(s, t, E, \mu) \, [\bar{\mathcal{X}}(0) \dots \mathcal{X}(s\bar{n})] \, [\bar{\mathcal{Q}}_s(tn) \dots \mathcal{H}(0)] \,. \tag{18}$$

At leading order in λ, operators can also contain insertions of transverse derivatives or gauge fields between the collinear or soft quark fields. Since $\partial_\perp^\mu \sim \lambda$ and $\mathcal{A}_{c\perp}^\mu \sim \mathcal{A}_{s\perp}^\mu \sim \lambda$, such transverse insertions must be accompanied by a factor of $\not{n}/in \cdot \partial_s \sim \lambda^{-1}$. The inverse derivative operator acts on a light soft field and implies an integration over the position of that field on the n light-cone, the effect of which can be absorbed into the Wilson coefficient functions. The appearance of $\not{n}$ in the numerator is enforced by reparameterization invariance. Using arguments along the lines of [16, 24] it then follows that in our case only single insertions of transverse objects are allowed. Finally, the multipole expansion of the soft fields in SCET$_{\rm I}$ ensures that the component $\bar{n} \cdot p_s$ of soft momenta does not enter Feynman diagrams at leading power. Likewise, at leading power there are no operators that contain $in \cdot \partial_c \sim \lambda^2$ (acting on collinear fields) or $n \cdot \mathcal{A}_c \sim \lambda^2$, since these are always suppressed with respect to the corresponding transverse quantities. In the presence of additional transverse derivatives or gluon fields the above argument about the cancellation of soft-collinear Wilson lines under the field redefinition (13) remains true, so that the resulting operators can again be expressed in terms of gauge-invariant building blocks. The new structures containing gluon fields are of the form

$$\int dr \, ds \, dt \, \widetilde{D}_k(r, s, t, E, \mu) \, [\bar{\mathcal{X}}(0) \dots \mathcal{A}_{c\perp}^\mu(r\bar{n}) \dots \mathcal{X}(s\bar{n})] \, [\bar{\mathcal{Q}}_s(tn) \dots \mathcal{H}(0)] \,,$$

$$\int ds \, dt \, du \, \widetilde{D}_k(s, t, u, E, \mu) \, [\bar{\mathcal{X}}(0) \dots \mathcal{X}(s\bar{n})] \, [\bar{\mathcal{Q}}_s(tn) \dots \mathcal{A}_{s\perp}^\mu(un) \dots \mathcal{H}(0)] \,, \tag{19}$$

and we define the corresponding Fourier-transformed coefficient functions as

$$D_k(\omega, u_2, u_3, E, \mu) = \int dr \, ds \, e^{2iE(u_2 s + u_3 r)} \int dt \, e^{-i\omega t} \, \widetilde{D}_k(r, s, t, E, \mu) \,,$$

$$D_k(\omega_1, \omega_2, u_2, E, \mu) = \int ds \, e^{2iEu_2 s} \int dt \, du \, e^{-i(\omega_1 t + \omega_2 u)} \, \widetilde{D}_k(s, t, u, E, \mu) \,. \tag{20}$$

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In matching calculations, u_3 is identified with the longitudinal momentum fraction x_3 carried by a final-state collinear gluon, while ω_1 and ω_2 are associated with the components $n \cdot l_q$ and $n \cdot l_g$ of the incoming soft anti-quark and gluon momenta.

We are now in a position to construct a basis of four-quark operators relevant to the discussion of the universal functions ζ_M in (4). It is convenient *not* to use the relation $\not{v} h = h$ in the matching from SCET$_\mathrm{I}$ onto SCET$_\mathrm{II}$, since then it is guaranteed that $\not{n}$ must appear to the left of the heavy-quark field $\mathcal{H}$. (This follows from the Fierz relation (24) below, where in our case M commutes with $\not{n}$.) The Feynman rules of SCET imply that the resulting operators must contain an odd number of Dirac matrices besides the matrix Γ_M contained in the effective current operator $J_M^{(0)}$. These operators must transform like the current $J_M^{(0)}$ under Lorentz transformations. Also, the soft and collinear parts of the four-quark operators must have non-zero projections onto the B meson and the final-state meson M. We set the transverse momenta of the mesons to zero, in which case there is no need to include operators with transverse derivatives acting on the products of all soft or collinear fields.

Case $M = P$: The resulting operators must transform as a scalar ($\Gamma_M = 1$). A basis of such operators is

$$O_1^{(P)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} \gamma_5 \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \frac{\not{\bar{n}}\not{n}}{4} \gamma_5 \mathcal{H}(0)\right],$$

$$O_2^{(P)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} \gamma_5 \, i\not{\partial}_\perp \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \frac{\not{n}}{2} \gamma_5 \mathcal{H}(0)\right],$$

$$O_3^{(P)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} \gamma_5 \, \mathcal{A}_{c\perp}(r\bar{n}) \, \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \frac{\not{n}}{2} \gamma_5 \mathcal{H}(0)\right],$$

$$O_4^{(P)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} \gamma_5 \, \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \, \mathcal{A}_{s\perp}(un) \frac{\not{n}}{2} \gamma_5 \mathcal{H}(0)\right].$$

(21)

The soft and collinear currents both transform like a pseudoscalar.

Case $M = V_\parallel$: The resulting operators must transform as a pseudo-scalar ($\Gamma_M = \gamma_5$). A basis of such operators is obtained by omitting the γ_5 between the two collinear spinor fields in (21), so that the collinear currents transform like a scalar. The Wilson coefficients for the cases $M = P$ and $M = V_\parallel$ coincide up to an overall sign due to parity invariance.

Case $M = V_\perp$: The resulting operators must transform as a transverse vector ($\Gamma_M = \gamma_{\perp\nu}$). A basis of such operators is

$$O_1^{(V_\perp)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} \gamma_{\perp\nu}\gamma_5 \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \frac{\not{\bar{n}}\not{n}}{4} \gamma_5 \mathcal{H}(0)\right],$$

$$O_2^{(V_\perp)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} (i\epsilon_{\nu\alpha}^\perp - g_{\nu\alpha}^\perp \gamma_5) \, i\partial_\perp^\alpha \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \frac{\not{n}}{2} \gamma_5 \mathcal{H}(0)\right],$$

$$O_3^{(V_\perp)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} (i\epsilon_{\nu\alpha}^\perp - g_{\nu\alpha}^\perp \gamma_5) \, \mathcal{A}_{c\perp}^\alpha(r\bar{n}) \, \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \frac{\not{n}}{2} \gamma_5 \mathcal{H}(0)\right],$$

$$O_4^{(V_\perp)} = g^2 \left[\bar{\mathcal{X}}(0) \frac{\not{\bar{n}}}{2} \gamma_{\perp\nu}\gamma_5 \mathcal{X}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_s(tn) \, \mathcal{A}_{s\perp}(un) \frac{\not{n}}{2} \gamma_5 \mathcal{H}(0)\right].$$

(22)

The soft currents transform like a pseudoscalar, while the collinear currents transform like a transverse axial-vector. (The relative sign between the two terms in $O_{2,3}^{(V_\perp)}$ can be determined by considering left and right-handed spinors $\bar{\mathcal{X}}$ and using relation (25) below.)

It is not necessary to include operators with a derivative on the soft spectator anti-quark. All such operators would contain $\bar{Q}_s\,(-i\,\overset{\leftarrow}{\partial}_\perp)\,\slashed{n}$, which can be related to a linear combination of the operators $O_1^{(M)}$ and $O_4^{(M)}$ using the equation of motion for the light-quark field. In our definitions above we have included the QCD coupling constant into the operators rather than the Wilson coefficient functions. This is natural, since a factor g^2 (not α_s/π) already arises in tree-level matching. More importantly, as we will see the matrix elements of the operators $O_i^{(M)}$ are *not* dominated by physics at the hard-collinear scale, but rather by long-distance hadronic physics. The coupling should therefore be evaluated at a low scale $\mu \ll \sqrt{E\Lambda}$.

3.1.1 Wilson coefficients of four-quark operators

We now calculate the momentum-space Wilson coefficients of the operators $O_i^{(M)}$ at tree level. The relevant graphs must contain a hard-collinear gluon exchange, which turns the soft B-meson spectator anti-quark into a collinear parton that can be absorbed by the final-state hadron. These hard-collinear gluons are integrated out when $\mathrm{SCET_I}$ is matched onto $\mathrm{SCET_{II}}$. However, at $O(\alpha_s)$ we can obtain the Wilson coefficients in $\mathrm{SCET_{II}}$ by directly matching QCD amplitudes onto the low-energy theory, without going through an intermediate effective theory.

As we have seen, at leading power the universal form factors ζ_M receive contributions from ordinary four-quark operators as well as from four-quark operators containing an additional collinear or soft gluon field. We start with a discussion of the matching calculation for the operators $O_{1,2}^{(M)}$, whose coefficients can be obtained by analyzing the diagrams shown in Figure 1 in the kinematic region where the outgoing quarks are collinear and the incoming quarks are soft. The resulting expression for the amplitude in the full theory has already been given in (3). Note that, by assumption, $n \cdot l \sim \Lambda$ and $x_2 \sim 1$, so that the result is well defined and it is consistent to neglect subleading terms in the gluon propagators.

We wish to express the result for the QCD amplitude as a combination of $\mathrm{SCET_{II}}$ matrix elements multiplied by coefficient functions. To this end, we must first eliminate the superficially leading term in (3) using the equations of motion. Assigning momenta to the two outgoing collinear lines as shown in (2), it follows that

$$\gamma_\mu E\slashed{n}\,\Gamma * \gamma^\mu = \frac{\slashed{p}_\perp}{x_1}\,\gamma_\mu\Gamma * \gamma^\mu - 2\Gamma * \frac{\slashed{p}_\perp}{x_2} + \dots\,, \tag{23}$$

where the dots denote power-suppressed terms. In the next step we use a Fierz transformation to recast the amplitude into a form that is convenient for our analysis. Taking into account that between collinear spinors the Dirac basis contains only three independent matrices, we find for general matrices M and N

$$2(\bar{u}_\xi M u_h)\,(\bar{v}_q N v_\xi) = (\bar{u}_\xi \frac{\overset{\leftarrow}{\slashed{n}}}{2} v_\xi)\,(\bar{v}_q N \frac{\slashed{n}}{2} M u_h) + (\bar{u}_\xi \frac{\overset{\leftarrow}{\slashed{n}}}{2}\gamma_5 v_\xi)\,(\bar{v}_q N \gamma_5 \frac{\slashed{n}}{2} M u_h)$$

$$+ (\bar{u}_\xi \frac{\overset{\leftarrow}{\slashed{n}}}{2}\gamma_{\perp\alpha} v_\xi)\,(\bar{v}_q N \gamma_\perp^\alpha \frac{\slashed{n}}{2} M u_h)\,. \tag{24}$$
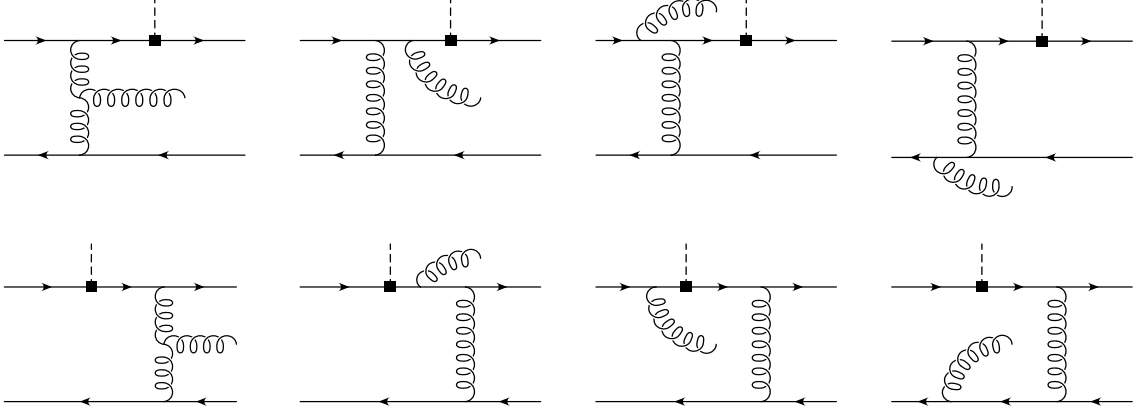
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Figure 2: Diagrams relevant to the matching calculation for operators containing an extra collinear gluon.

To simplify the Dirac algebra we use the identities

$$\gamma^\mu_\perp \gamma_5 \, \rlap{/}{n} = i\epsilon^{\mu\nu}_\perp \gamma_{\perp\nu} \, \rlap{/}{n} \,, \qquad \gamma^\mu_\perp \gamma^\nu_\perp \, \rlap{/}{n} = (g^{\mu\nu}_\perp - i\epsilon^{\mu\nu}_\perp \gamma_5) \, \rlap{/}{n} \,. \tag{25}$$

Finally, we include a minus sign from fermion exchange under the Fierz transformation and project the collinear and soft "currents" in the resulting expressions onto color-singlet states.

Once in this form, the different contributions to the amplitude can be readily identified with matrix elements of the operators $O^{(M)}_{1,2}$. For $M = P, V_\parallel$ we obtain

$$D^{(P)}_1 = -D^{(V_\parallel)}_1 = -\frac{C_F}{N} \frac{1 + u_2}{4E^2 u_2^2 \, \omega} \,, \qquad D^{(P)}_2 = -D^{(V_\parallel)}_2 = -\frac{C_F}{N} \frac{1}{4E^2 u_1 u_2^2 \, \omega^2} \,, \tag{26}$$

while for $M = V_\perp$ we find

$$D^{(V_\perp)}_1 = \frac{C_F}{N} \frac{1}{4E^2 u_2^2 \, \omega} \,, \qquad D^{(V_\perp)}_2 = \frac{C_F}{N} \frac{1}{4E^2 u_2^2 \, \omega^2} \,. \tag{27}$$

3.1.2 Wilson coefficients of four-quark operators with an extra gluon

The matching calculation for the operators $O^{(M)}_{3,4}$ proceeds along the same lines. However, in this case it is necessary to study diagrams with four external quarks and an external gluon, which we treat as a background field. Let us first discuss the case with three collinear particles in the final state. The relevant QCD graphs are shown in Figure 2. Physically, they correspond to non-valence Fock states of the light final-state meson, but this interpretation is irrelevant for the matching calculation, which can be done with free quark and gluon states.

By assumption, each of the three collinear particles carries large momentum components along the n direction. We denote the corresponding longitudinal momentum fractions by x_1 (quark), x_2 (anti-quark) and x_3 (gluon), where $x_1 + x_2 + x_3 = 1$. The calculation of the diagrams in Figure 2 exhibits that at leading power the amplitude depends only on the light-cone components $n \cdot l$ and $\bar{n} \cdot p_i = 2Ex_i$ of the external momenta, and that only transverse

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components of the external gluons fields must be kept. These observations are in accordance with the structure of the operator $O_3^{(M)}$. One must add to the diagrams shown in the figure a contribution arising from the application of the equation of motion for the collinear quark fields that led to (23). One way of obtaining it is to include diagrams with gluon emission from the external collinear quark lines in the matching calculation. Performing the Fierz transformation and projecting onto the color singlet-singlet operators $O_3^{(M)}$, we obtain the coefficient functions

$$
\begin{aligned}
D_3^{(P)} = -D_3^{(V_\parallel)} &= \frac{1}{8E^2(u_2+u_3)^2\omega^2}\left[\frac{u_3}{u_2} - 1 + \frac{2}{N^2} - \frac{2C_F}{N}\frac{(u_2+u_3)^2}{u_2(u_1+u_3)}\right], \\
D_3^{(V_\perp)} &= -\frac{1}{8E^2(u_2+u_3)^2\omega^2}\left[\frac{u_3}{u_2} - 1 + \frac{2}{N^2} + \frac{1}{N^2}\frac{(u_2+u_3)^2}{u_2(u_1+u_2)}\right].
\end{aligned}
\tag{28}
$$

Next, we calculate the contributions from diagrams with three external soft particles, which physically correspond to three-particle Fock states of the B meson. Again there are eight diagrams, analogous to those in Figure 2. The initial soft gluon is attached to either an off-shell intermediate line or a collinear line. The diagrams with external gluons must again be supplemented by a contribution resulting from the application of the equation of motion $\bar{v}_q \slashed{v} = O(g)$ used in the analysis of the four-quark amplitude in (3). After Fierz transformation and projection onto the color singlet-singlet operators $O_4^{(M)}$, we find the coefficient functions

$$
\begin{aligned}
D_4^{(P)} = -D_4^{(V_\parallel)} &= \frac{1}{8E^2u_2^2(\omega_1+\omega_2)^2}\left[\left(1 - \frac{2C_F}{N}u_2\right)\frac{\omega_2}{\omega_1} + \frac{1}{N^2}\right], \\
D_4^{(V_\perp)} &= -\frac{1}{8E^2u_2^2(\omega_1+\omega_2)^2}\left[\left(1 + \frac{1}{N^2}\frac{u_2}{u_1}\right)\frac{\omega_2}{\omega_1} + \frac{1}{N^2}\frac{1}{u_1}\right].
\end{aligned}
\tag{29}
$$

The variables ω_1 and ω_2 correspond to the light-cone components $n\cdot l_q$ and $n\cdot l_g$ of the incoming momenta of the soft anti-quark and gluon, respectively.

3.1.3 Endpoint singularities

The Wilson coefficients of the SCET_II four-quark operators become singular in the limit where some of the momentum components of the external particles tend to zero. When the $B \to M$ matrix elements of the operators $O_k^{(M)}$ are evaluated, these singularities give rise to endpoint divergences of the resulting convolution integrals with LCDAs. For instance, the matrix elements of the operators $O_1^{(M)}$ and $O_4^{(M)}$ involve the leading twist-2 projection onto the light meson M. The corresponding LCDAs ϕ_M are expected to vanish linearly as $u_2 \to 0$, whereas the corresponding coefficients contain terms that grow like $1/u_2^2$, giving rise to logarithmically divergent convolution integrals. Similarly, the operators $O_2^{(M)}$ and $O_3^{(M)}$ involve the leading-order projection onto the B-meson LCDA $\phi_B^{(+)}$, which is expected to vanish linearly as $\omega \to 0$ [25]. Once again, logarithmic singularities arise because the corresponding coefficients contain terms that grow like $1/\omega^2$.

While the logarithmic divergences in the convolution integrals could be avoided by introducing some infra-red regulators, they indicate that leading-order contributions to the amplitudes arise from momentum regions that cannot be described correctly in terms of collinear

or soft fields. In Section 4, we will explain that in SCET_{II} these configurations are accounted for by matrix elements of operators containing the soft-collinear messenger fields.

It is interesting to note that the operators $O_{3,4}^{(M)}$ arise from the matching of the SCET_{I} operator called T_0^{F} in [17] onto operators of the low-energy effective theory. The fact that the matrix elements of these operators contain endpoint divergences shows that the criterion of hard-collinear/soft factorization of SCET_{I} operators is insufficient to decide whether matrix elements can be factorized into separate soft and collinear entities in SCET_{II} or not.

3.2 Spin-symmetry breaking contributions

The two-particle amplitude in (3) also includes contributions for which the Dirac structure is different from $\not{n}\,\Gamma\,h$. These give rise to symmetry-breaking contributions to the form factors. We shall not derive a complete basis of all possible symmetry-breaking operators (there are many) but rather list the ones that enter at first order in α_s. Since the spin-symmetry violating terms can be factorized in the form of the second term in (4), they are associated with a short-distance coupling constant $\alpha_s(\sqrt{E\Lambda})$. It is therefore appropriate in this case to include the coupling constant in the Wilson coefficient functions.

Let Γ denote the Dirac structure of the flavor-changing currents $\bar{q}\,\Gamma\,b$, whose matrix elements define the form factors. To first order in α_s, the spin-symmetry breaking terms can then be obtained from the matrix elements of two operators given by

$$
Q_1 = [\bar{\mathfrak{X}}(0)\,\frac{\not{\bar{n}}}{2}\begin{pmatrix}1\\\gamma_5\\\gamma_{\perp\alpha}\end{pmatrix}\mathfrak{X}(s\bar{n})]\,[\bar{\mathfrak{Q}}_s(tn)\,\frac{\not{n}}{2}\,\gamma_\mu\begin{pmatrix}1\\-\gamma_5\\-\gamma_\perp^\alpha\end{pmatrix}\Gamma\,\gamma^\mu\not{n}\,\mathcal{H}(0)]\,,
$$

$$
Q_2 = [\bar{\mathfrak{X}}(0)\,\frac{\not{\bar{n}}}{2}\begin{pmatrix}1\\\gamma_5\\0\end{pmatrix}\mathfrak{X}(s\bar{n})]\,[\bar{\mathfrak{Q}}_s(tn)\,\frac{\not{n}\not{\bar{n}}}{4}\begin{pmatrix}1\\-\gamma_5\\0\end{pmatrix}\Gamma\,\mathcal{H}(0)]\,,
$$

$$(30)$$

where each line contributes for a different final-state meson M. The corresponding Wilson coefficients are

$$
\hat{T}_1 = -\frac{C_F}{N}\,\frac{\pi\alpha_s}{2m_B E u_2\,\omega}\,, \qquad \hat{T}_2 = \frac{C_F}{N}\,\frac{\pi\alpha_s}{E^2 u_2\,\omega}\,. \tag{31}
$$

Linear combinations of $\hat{T}_1$ and $\hat{T}_2$ determine (up to prefactors) the hard-scattering kernels T_i in (4). The matrix elements of the operators $Q_{1,2}$ can be expressed in terms of the leading-order LCDAs of the B meson and the light meson M. Only the B-meson LCDA called $\phi_B^{(+)}$ contributes [27] because of the factor $\not{n}$ next to $\bar{Q}_s$. This property holds true to all orders in perturbation theory [17]. The resulting convolution integrals are convergent. Evaluating these matrix elements we reproduce the spin-symmetry breaking terms obtained in [20].

4   Physics of endpoint singularities – a toy model

Our strategy in the previous sections has been to perform matching calculations by expanding QCD amplitudes in powers of Λ/E, assuming that collinear and soft external momenta
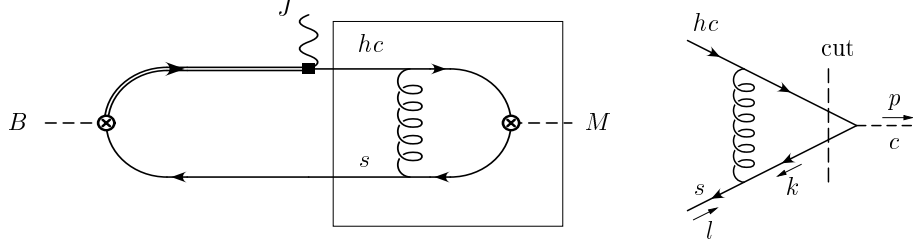
Figure 3: Triangle subgraph whose spectral function can be used to study endpoint singularities on the collinear side.

have the scaling assigned to them in SCET. We have then matched the results onto SCET$_{\rm II}$ operators containing soft and collinear fields and read off the corresponding Wilson coefficient functions in momentum space. A problematic aspect of this procedure has been the observation that, if the matrix elements of the effective-theory operators are expressed in terms of meson LCDAs, then the resulting convolution integrals do not converge. Endpoint singularities arise, which correspond to exceptional momentum configurations in which some of the partons inside the external hadrons carry very small momentum. The question naturally arises how one should interpret these singularities, and whether the results we found for the short-distance coefficient functions are in fact correct.

The fact that endpoint configurations are not kinematically suppressed points to the relevance of new momentum modes. In the limit $x_2 \to 0$, the scaling associated with the collinear anti-quark in the final state of Figure 1 changes from $(\lambda^2, 1, \lambda)$ to $(\lambda^2, \lambda, \dots)$. Likewise, in the limit $n \cdot l \to 0$, the scaling associated with the soft spectator anti-quark in the initial state changes from $(\lambda, \lambda, \lambda)$ to $(\lambda^2, \lambda, \dots)$. The soft-collinear messenger fields in the SCET$_{\rm II}$ Lagrangian have precisely the scaling properties corresponding to these exceptional configurations. (Since the modes in an effective theory are always on-shell, the transverse components of soft-collinear momenta scale like $\lambda^{3/2}$. We will see below that this is not really relevant.)

The purpose of this section is to analyze the interplay of the various modes present in SCET$_{\rm II}$ and to see how soft-collinear messengers are connected with the phenomenon of endpoint singularities. We will do this with the help of a toy example. Consider the triangle subgraph enclosed by the box in the diagram shown on the left-hand side in Figure 3, which is one of the two gluon-exchange graphs relevant to the $B \to M$ form factors (see Figure 1). To understand the physics of endpoint singularities we focus on the side of the light meson M (an analogous discussion could be given for the B-meson side). We study the discontinuity of the triangle diagram in the external collinear momentum p^2. The resulting spectral density $\varrho(p^2)$ models the continuum of light final-state hadrons that can be produced on the collinear side. We will not bother to project out a particular light meson from this spectral density. For simplicity, we will also ignore any numerator structure in the triangle subgraph and instead study the corresponding scalar triangle, which has already been discussed in [22]. The results we obtain are nevertheless general.

Let us define the discontinuity

$$\frac{1}{2\pi i} \left[I(p^2 + i0) - I(p^2 - i0) \right] \equiv D \cdot \theta(p^2) \tag{32}$$

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of the scalar triangle integral

$$I = i\pi^{-d/2}\mu^{4-d} \int d^d k \, \frac{2l_+ \cdot p_-}{(k^2 + i0)\,[(k+l)^2 + i0]\,[(k+p)^2 + i0]} \tag{33}$$

in $d = 4 - 2\epsilon$ space-time dimensions, where p is the outgoing collinear momentum and l the incoming soft momentum. It will be convenient to define the invariants $L^2 \equiv -l^2 - i0$ and $Q^2 \equiv -(l-p)^2 = 2l_+ \cdot p_- + \dots$, which scale like $L^2 \sim \lambda^2$ and $Q^2 \sim \lambda$. (In physical units, $L^2 \sim \Lambda^2$ and $Q^2 \sim E\Lambda$ with $E \gg \Lambda$.) We will assume that these quantities are non-zero. From [22], we can obtain explicit results for the discontinuity D and for the various regions of loop momentum k that give a non-vanishing contribution. We find that

$$D = \ln \frac{Q^2}{L^2} + O(\epsilon, \lambda), \tag{34}$$

and that the momentum configurations that contribute to this result are those where the loop momentum is either collinear, meaning that $k \sim (\lambda^2, 1, \lambda)$, or soft-collinear, meaning that $k \sim (\lambda^2, \lambda, \lambda^{3/2})$. The contributions from these two regions are

$$
\begin{aligned}
D_{\mathrm{C}} &= \frac{\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{\mu^2}{p^2}\right)^{\epsilon} = -\frac{1}{\epsilon} + \gamma_E - \ln \frac{\mu^2}{p^2} + O(\epsilon), \\
D_{\mathrm{SC}} &= \Gamma(\epsilon) \left(\frac{\mu^2 Q^2}{p^2 L^2}\right)^{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \ln \frac{\mu^2 Q^2}{p^2 L^2} + O(\epsilon),
\end{aligned}
\tag{35}
$$

which add up to the correct answer.

It is instructive to rewrite these results in a more transparent form. To this end we use Cutkosky rules to evaluate the discontinuities of the diagrams directly and perform all phase-space integrations except the integral over the light-cone component $\bar{n} \cdot k$ of the loop momentum, which we parameterize as $\bar{n} \cdot k \equiv -x_2 \bar{n} \cdot p$. As in previous sections, x_2 denotes the fraction of longitudinal momentum carried by the anti-quark in the final-state. The exact result for the discontinuity of the scalar triangle is ($\bar{x}_2 \equiv 1 - x_2$)

$$
\begin{aligned}
D &= \frac{1}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{p^2}\right)^{\epsilon} \int_0^1 dx_2 \, \frac{(x_2 \bar{x}_2)^{-\epsilon}}{x_2 + L^2/Q^2} + O(\lambda) \\
&= \int_0^1 dx_2 \, \frac{1}{x_2 + L^2/Q^2} + O(\epsilon, \lambda) = \int_{L^2/Q^2}^1 \frac{dx_2}{x_2} + O(\epsilon, \lambda).
\end{aligned}
\tag{36}
$$

The collinear and soft-collinear contributions separately are divergent even though they correspond to tree diagrams (after the two propagators have been cut). We obtain

$$
\begin{aligned}
D_{\mathrm{C}} &= \frac{1}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{p^2}\right)^{\epsilon} \int_0^1 \frac{dx_2}{x_2} \, (x_2 \bar{x}_2)^{-\epsilon}, \\
D_{\mathrm{SC}} &= \frac{1}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{p^2}\right)^{\epsilon} \int_0^\infty dx_2 \, \frac{x_2^{-\epsilon}}{x_2 + L^2/Q^2}.
\end{aligned}
\tag{37}
$$

The collinear contribution is infra-red singular for $x_2 \to 0$, which is an example of an endpoint singularity. In the present case, the singularity is regularized dimensionally by keeping ϵ non-zero. The soft-collinear contribution is infra-red finite but ultra-violet divergent, since x_2 runs up to ∞. Again, this divergence is regularized dimensionally. Evaluating the remaining integrals one recovers the exact results in (35).

Several comments are in order:

i) The result for the spectral density in the full theory is finite. The endpoint singularity is regularized by keeping subleading terms in the hard-collinear propagator $1/[-(k+l)^2] \simeq 1/(x_2 Q^2 + L^2)$. When the subleading terms are dropped based on naive power counting (as we did in the analysis of the previous sections) the full-theory result reduces to the contribution obtained from the collinear region. An endpoint singularity arises in this case, which however can be regularized dimensionally. In order for this to happen in the realistic case of external meson states, it would be necessary to perform the projections onto the meson LCDAs in $d \neq 4$ dimensions. The factor $(x_2 \bar{x}_2)^{-\epsilon}$ in (37) would then correspond to a modification of the LCDAs, which renders the convolution integrals finite.

ii) The collinear approximation fails for values $x_2 = O(\lambda)$. For such small momentum fractions the exact full-theory result is reproduced by the contribution obtained from the soft-collinear region. In fact, the collinear and soft-collinear contributions in (37) coincide with the first terms in the Taylor expansion of the full-theory result in (36) in the limits where $x_2 = O(1) \gg L^2/Q^2$ and $x_2 = O(\lambda) \ll 1$. The fact that x_2 runs up to ∞ in the soft-collinear contribution is not a problem. In dimensional regularization the integral receives significant contributions only from the region where $x_2 \sim L^2/Q^2 = O(\lambda)$. To see this, one can introduce a cutoff δ to separate the collinear and soft-collinear contributions, chosen such that $1 \gg \delta \gg \lambda$. This cutoff is introduced as an infra-red regulator in the collinear integral ($\int_0^1 \to \int_\delta^1$) and as an ultra-violet regulator in the soft-collinear integral ($\int_0^\infty \to \int_0^\delta$). The integrals can then be evaluated setting $\epsilon \to 0$. This yields $D_{\mathrm{C}} = -\ln \delta$ and $D_{\mathrm{SC}} = \ln \delta + \ln \frac{Q^2}{L^2} + O(\lambda/\delta)$. The sum of the two contributions once again reproduces the exact result.

iii) Next, note that the soft-collinear contribution *precisely* reproduces the endpoint behavior of the full-theory amplitude. It is irrelevant in this context that the soft-collinear virtuality $k_{sc}^2 \sim E^2 \lambda^3$ is parametrically smaller than the QCD scale Λ^2. What matters is that the plus and minus components of the soft-collinear momentum, $k_{sc} \sim (\lambda^2, \lambda, \dots)$, are of the same order as the corresponding components of a collinear momentum in the endpoint region, where $\bar{n} \cdot p \sim \lambda$ rather than being $O(1)$.

iv) Finally, we see that the endpoint divergences we encountered in the Section 3.1.3 were not regularized because we dropped the dimensional regulator when performing the projections onto meson LCDAs. This, however, does not affect the results for the Wilson coefficients of the SCET$_{\mathrm{II}}$ operators, which remain valid. In the toy model, the collinear contribution is represented in the effective theory as the discontinuity of the integral (corresponding to the first diagram on the right-hand side in Figure 4)

$$
\begin{aligned}
I_{\mathrm{C}} &= i\pi^{-d/2} \mu^{4-d} \int d^d k \, \frac{2l_+ \cdot p_-}{(k^2 + i0)\,(2k_- \cdot l_+)\,[(k+p)^2 + i0]} \\
&= i\pi^{-d/2} \mu^{4-d} \int d^d k \, \frac{(-1)}{x_2} \frac{1}{(k^2 + i0)\,[(k+p)^2 + i0]}\,,
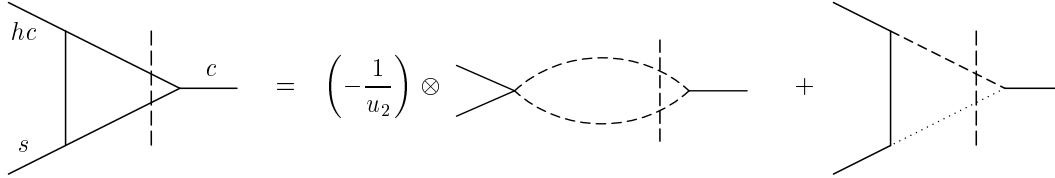\end{aligned}
\tag{38}
$$

Figure 4: Collinear and soft-collinear contributions to the scalar triangle graph in SCET$_{\mathrm{II}}$. Dashed (dotted) lines represent collinear (soft-collinear) propagators.

which shows that the Wilson coefficient resulting from integrating out the hard-collinear propagator is simply $-1/u_2$.

To summarize this discussion, we stress that to reproduce the behavior of the full amplitude it is necessary to include in the effective theory contributions involving collinear fields and those involving soft-collinear messengers, as indicated in Figure 4. The latter ones represent the exact behavior of the amplitude in the endpoint region. Whether or not endpoint configurations contribute at leading order in power counting is equivalent to the question of whether or not operators involving soft-collinear fields can arise at leading power. This shows the power of our formalism. Operators containing soft-collinear fields can be used to parameterize in a systematic way the long-distance physics associated with endpoint configurations in the external mesons states. This will be discussed in more detail in the next section. It also follows that the scaling properties of operators containing soft-collinear fields can be used to make model-independent statements about the convergence of convolution integrals that arise in QCD factorization theorems such as (4). We will exploit this connection in Section 7.

5   Soft-collinear messengers and the soft overlap

In Section 3 we have derived four-quark operators built out of collinear and soft fields, which contribute at leading order to the universal soft functions ζ_M in (4). Power counting shows that the products of these operators with their Wilson coefficients scale like λ^4. (Here one uses that $dr \sim ds \sim 1$ and $dt \sim du \sim \lambda^{-1}$.) Taking into account the counting of the external hadron states, $|B\rangle \sim \lambda^{-3/2}$ and $\langle M| \sim \lambda^{-1}$, it follows that the corresponding contributions to the universal form factors scale like $\lambda^{3/2}$. In other words, heavy-to-light form factors are quantities that vanish at leading order in the large-energy expansion. The fact that the resulting convolution integrals were infra-red singular suggested that there should be other contributions of the same order in power counting, which cannot be described in terms of collinear or soft fields.

An important observation made in the previous section was that the endpoint behavior of QCD amplitudes is described in the low-energy theory in terms of diagrams involving soft-collinear messenger fields. For instance, in the last graph in Figure 4 the soft spectator anti-quark inside the B meson turns into a soft-collinear anti-quark by the emission of a soft gluon. The soft-collinear anti-quark is then absorbed by the final-state meson. In a similar way, endpoint singularities on the B-meson side would correspond to a situation where the

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initial state contains a soft-collinear spectator anti-quark, which absorbs a collinear gluon and turns into a collinear anti-quark. The SCET$_{\rm II}$ Lagrangian contains the corresponding interaction terms only at subleading order in λ. However, because the universal form factors ζ_M themselves are power-suppressed quantities, these subleading interactions will nevertheless give rise to leading-power contributions.

We will need the first two non-vanishing orders in the interactions that couple a soft-collinear quark to a soft or collinear quark. The results are most transparent when expressed in terms of the gauge-invariant building blocks introduced in (13). They are [22]

$$\mathcal{L}_{\bar{q}\theta}^{(1/2)} = \bar{\mathcal{Q}}_s \, \mathcal{A}_{s\perp} W_{sc}^\dagger \, \theta \,, \qquad \mathcal{L}_{\bar{\theta}\xi}^{(1/2)} = \bar{\sigma} \, S_{sc} \mathcal{A}_{c\perp} \, \mathcal{X} \,, \tag{39}$$

and

$$\mathcal{L}_{\bar{q}\theta}^{(1)} = \bar{\mathcal{Q}}_s \, \mathcal{A}_{s\perp} W_{sc}^\dagger \, (x_\perp \cdot D_{sc} \, \theta + \sigma) + \bar{\mathcal{Q}}_s \, \frac{\slashed{n}}{2} \, \bar{n} \cdot \mathcal{A}_s \, W_{sc}^\dagger \, \sigma \,,$$
$$\mathcal{L}_{\bar{\theta}\xi}^{(1)} = \bar{\sigma} \, x_\perp \cdot \overleftarrow{D}_{sc} \, S_{sc} \mathcal{A}_{c\perp} \, \mathcal{X} - \bar{\theta} \, S_{sc} \mathcal{A}_{c\perp} \, \frac{\slashed{n}}{2} \, \frac{1}{i\bar{n} \cdot \partial} \, (i\slashed{\partial}_\perp + \mathcal{A}_{c\perp}) \, \mathcal{X} + \bar{\theta} \, S_{sc} \, \frac{\slashed{n}}{2} \, n \cdot \mathcal{A}_c \, \mathcal{X} \,, \tag{40}$$

where

$$\sigma = -\frac{\slashed{n}}{2} \, \frac{1}{i\bar{n} \cdot D_{sc}} \, i\slashed{D}_{sc\perp} \, \theta \tag{41}$$

contains the "small components" of the soft-collinear quark field, which are integrated out in the construction of the effective Lagrangian. The longitudinal components of the calligraphic gluon fields are defined in [22]. (Note also that $n \cdot \mathcal{A}_s = 0$ and $\bar{n} \cdot \mathcal{A}_c = 0$.) The soft-collinear fields enter these results in combinations such as $W_{sc}^\dagger \theta$ or $S_{sc}^\dagger \theta$, which are gauge invariant. Soft and collinear fields live at position x, while soft-collinear fields are evaluated at position x_+ for $\mathcal{L}_{\bar{q}\theta}$ and x_- for $\mathcal{L}_{\bar{\theta}\xi}$. The measure d^4x associated with these interactions scales like λ^{-4}. The superscript on the Lagrangians indicates at which order in power counting ($\lambda^{1/2}$ or λ) the corresponding terms contribute to the action.

Next, we need the representation of the flavor-changing SCET$_{\rm I}$ current $J_M^{(0)}$ in (14) in terms of operators in the low-energy theory SCET$_{\rm II}$. As shown in [23], at leading power, and at the matching scale $\mu = \mu_{hc}$, the relation reads

$$J_M^{(0)}(x) \to \mathcal{J}_M^{(0)}(x) = \bar{\mathcal{X}}(x_+ + x_\perp) \, \Gamma_M \, (S_{sc}^\dagger W_{sc})(0) \, \mathcal{H}(x_- + x_\perp) \tag{42}$$

with a Wilson coefficient equal to unity. The anomalous dimensions of the currents are the same in the two theories. The product $(S_{sc}^\dagger W_{sc})(0)$ arises since soft-collinear messenger fields cannot be decoupled from the current operator $\mathcal{J}_M^{(0)}$ in SCET$_{\rm II}$, contrary to the case of the color singlet-singlet four-quark operators discussed in Section 3.

Using these results, we can write down a tri-local operator whose matrix element provides a long-distance contribution to the universal form factors. It is

$$O_5^{(M)} = i^2 \int d^4x \, d^4y \, {\rm T} \left\{ \mathcal{L}_{\bar{q}\theta}^{(1/2)}(x) \, \mathcal{L}_{\bar{\theta}\xi}^{(1)}(y) \, \mathcal{J}_M^{(0)}(0) + \mathcal{L}_{\bar{q}\theta}^{(1)}(x) \, \mathcal{L}_{\bar{\theta}\xi}^{(1/2)}(y) \, \mathcal{J}_M^{(0)}(0) \right\}. \tag{43}$$

Even after the decoupling transformation (13) this operator contains arbitrarily complicated soft-collinear exchanges, as illustrated in the cartoon in Figure 5.
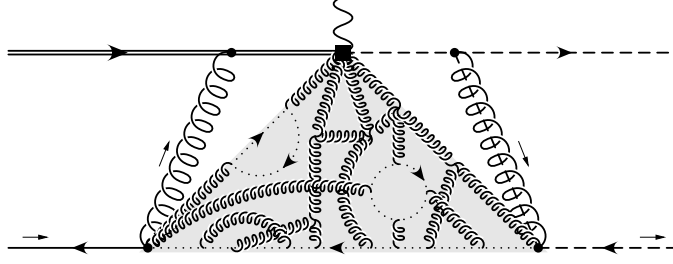
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Figure 5: An artist's view of the soft-collinear messenger contribution to the form factors. The shaded region contains soft-collinear interactions. The arrows indicate the flow of the components $n \cdot p_s$ (left) and $\bar{n} \cdot p_c$ (right) of soft and collinear momenta, which do not enter the soft-collinear block.

Note that the superficially leading term in the time-ordered product cancels [22]. This can be understood as follows. After the decoupling transformation the strong-interaction part of the SCET$_{\mathrm{II}}$ Lagrangian no longer contains unsuppressed interactions between soft-collinear messengers and soft or collinear fields. In order to preserve a transparent power counting it is then convenient to define hadron states in the effective theory as eigenstates of one of the two leading-order Lagrangians $\mathcal{L}_s$ and $\mathcal{L}_c$. For instance, we define a "SCET pion" to be a bound state of only collinear fields, and a "SCET B meson" to be a bound state of only soft fields. The SCET pion state coincides with the true pion, because the collinear Lagrangian is equivalent to the QCD Lagrangian [15].[3] The SCET B meson coincides with the asymptotic heavy-meson state as defined in heavy-quark effective theory. It is important in this context that time-ordered products of soft-collinear fields with only collinear or only soft fields vanish to all orders as a consequence of analyticity [22]. Hence, soft-collinear modes do not affect the spectrum of hadronic eigenstates of the collinear or soft Lagrangians. Once the SCET$_{\mathrm{II}}$ hadron states are defined in this way, each term in the time-ordered product (43) can be factorized into a part containing all soft and collinear fields times a vacuum correlation function of the soft-collinear fields. These vacuum correlators must be invariant under rotations in the transverse plane. It follows that the correlator arising from the superficially leading term in the time-ordered product vanishes, since

$$\langle \Omega | \, \mathrm{T} \left\{ (\bar{\sigma} \, S_{sc})_i (y_-) \, (S_{sc}^\dagger W_{sc})_{jk}(0) \, (W_{sc}^\dagger \theta)_l(x_+) \right\} | \Omega \rangle \tag{44}$$

contains a single transverse derivative (see (41)).

The leading terms in the time-ordered product in (43) scale like λ^4, since $\mathcal{J}_M^{(0)} \sim \lambda^{5/2}$. They thus contribute at the same order in power counting to the universal functions ζ_M as the four-quark operators discussed in Section 3. When $O_5^{(M)}$ is added to the list of four-quark operators a complete description of the soft-overlap contribution is obtained. The Wilson coefficient of this new operator follows from the fact that there is no non-trivial matching

---

[3]The endpoint region of the pion wave function is not described in terms of a Fock component containing a soft-collinear parton, but rather in terms of a time-ordered product of the SCET pion state with an insertion of the Lagrangian $\mathcal{L}_{\bar{\theta}\xi}$. This insertion is non-zero only in processes where also soft partons are involved.

coefficient in (42), that the current operators have the same anomalous dimensions in SCET_I and SCET_II, and that the SCET_II Lagrangian is not renormalized. Hence, to all orders in perturbation theory

$$D_5^{(M)}(\mu_{hc}, \mu) = \frac{C_i(\mu)}{C_i(\mu_{hc})} \equiv D_5(\mu_{hc}, \mu)\,, \tag{45}$$

which is in fact a universal function, independent of the labels "i" and "M". As before, μ_{hc} denotes the hard-collinear matching scale, at which the transition $\text{SCET}_\text{I} \to \text{SCET}_\text{II}$ is made. The appearance of $C_i(\mu_{hc})$ in the denominator of this relation is due to the fact that this coefficient was factored out in the definition of $D_k^{(M)}$, see (4) and (6).

If we define by $\xi_k^{(M)}$ the $B \to M$ hadronic matrix elements of the operators $O_k^{(M)}$, then the sum $\zeta_M = \sum_k D_k^{(M)} \xi_k^{(M)}$ describes the entire soft overlap contribution. Each term in this sum gives a contribution of order $\lambda^{3/2}$ to the form factors, in accordance with the scaling law obtained a long time ago in the context of QCD sum rules [28]. We have thus completed the derivation of the new factorization formula (6). Whereas the Wilson coefficients $D_k^{(M)}$ depend on the renormalization scale μ as well as on the hard-collinear scale $\mu_{hc} \sim \sqrt{E\Lambda}$, the characteristic scale of the hadronic matrix elements $\xi_k^{(M)}$ is the QCD scale Λ, not the hard-collinear scale. While this is obvious for the matrix elements of the operator $O_5^{(M)}$, it also holds true for the remaining matrix elements, for which the sensitivity to long-distance physics is signaled by the presence of endpoint singularities. It remains to discuss how our results are affected by single and double logarithmic corrections arising in higher orders of perturbation theory.

6   Operator mixing and Sudakov logarithms

Because the operators $O_k^{(M)}$ share the same global quantum numbers they can mix under renormalization. This mixing is governed by a 5×5 matrix of anomalous dimension kernels, which are in general complicated functions of the light-cone variables u_i and ω_i. The anomalous dimension matrix governing this mixing has the structure

$$\boldsymbol{\gamma} = \left(\begin{array}{cccc|c} \gamma_{11} & 0 & 0 & \gamma_{14} & 0 \\ 0 & \gamma_{22} & \gamma_{23} & 0 & 0 \\ 0 & \gamma_{32} & \gamma_{33} & 0 & 0 \\ \gamma_{41} & 0 & 0 & \gamma_{44} & 0 \\ \hline \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} \end{array}\right). \tag{46}$$

Because the operators $O_{1\ldots4}^{(M)}$ consist of products of soft and collinear currents (after decoupling of soft-collinear messengers), the entries in the upper left 4×4 sub-matrix can all be written as sums of soft and collinear anomalous dimensions for the corresponding current operators. We have taken into account that the operators $O_2^{(M)}$ and $O_3^{(M)}$ mix under renormalization (this mixing is determined by the mixing of twist-3 two-particle and three-particle LCDAs for light mesons), as do the operators $O_1^{(M)}$ and $O_4^{(M)}$ (this mixing has not yet been worked

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out but is allowed on general grounds). The operator $O_5^{(M)}$ consisting of a triple time-ordered product requires the "local" operators as counter-terms; however, those operators do not mix back into $O_5^{(M)}$.

Because of the structure of the anomalous dimension matrix it follows that the coefficient γ_{55} is one of the eigenvalues, which governs the scale dependence of the Wilson coefficient D_5 in (45). This coefficient coincides with the well-known anomalous dimension of the current operators $J_M^{(0)}$ and $\mathcal{J}_M^{(0)}$ computed in [12, 23]. The corresponding "operator eigenvector" can be written as a linear combination (in the convolution sense)

$$\mathcal{O}_5^{(M)} = O_5^{(M)} + \sum_{k=1}^{4} d_k^{(M)} \otimes O_k^{(M)}, \qquad (47)$$

where the coefficients $d_k^{(M)}$ are independent of the high-energy matching scale μ_{hc}. The other four eigenvectors are linear combinations of the operators $O_k^{(M)}$ with $k \neq 5$. This observation has an important consequence: Since the $B \to M$ matrix elements of the operators $O_k^{(M)}$ with $k \neq 5$ are singular, only linear combinations of these operators with $O_5^{(M)}$ can contribute to the form factors, because only they could be free of such singularities.[4] However, the single combination of all five operators that is an eigenvector is $\mathcal{O}_5^{(M)}$. Therefore, we conclude that the combination of operators contributing to the universal form factors must, up to an overall factor, coincide with this eigenvector, and hence

$$d_k^{(M)} = \frac{D_k^{(M)}}{D_5} \qquad (48)$$

to all orders in perturbation theory. This result implies several remarkable facts, which could be checked by higher-order perturbative calculations. First, since the coefficients $d_k^{(M)}$ must be independent of μ_{hc}, each of the coefficients $D_k^{(M)}$ must share the same dependence on the high-energy matching scale, and this dependence must cancel that of the SCET$_{\rm I}$ Wilson coefficients $C_i(E, \mu_{hc})$ in (4). This statement is compatible with the observation that the operators $O_k^{(M)}$ arise from the matching of time-ordered products of the current $J_M^{(0)}$ with the SCET$_{\rm I}$ Lagrangian onto the low-energy theory [17], but it is by no means an automatic consequence of this observation. Secondly, it follows that there must be intimate relations between the Wilson coefficients $D_k^{(M)}$ with $k \neq 5$ and the off-diagonal elements γ_{5k} of the anomalous dimension matrix in (46), which are connected through the RG equation

$$\left(\frac{d}{d \ln \mu} + \gamma_{55} \right) d_k - d_j \otimes \gamma_{jk} = \gamma_{5k}, \qquad (49)$$

which follows from (47).

With a slight abuse of notation, let us now denote by ζ_M the $B \to M$ hadronic matrix element of the eigenvector $\mathcal{O}_5^{(M)}$ in SCET$_{\rm II}$. Combining (4) and (45), we then find that the

---

[4]Since each eigenvector is multiplied by a Wilson coefficient with a different dependence on the high-energy matching scale (as determined by its anomalous dimension eigenvalue), no accidental cancellations between different eigenvectors can occur.

spin-symmetric universal form-factor term can be rewritten as

$$C_i(E, \mu_{\mathrm{I}})\, \zeta_M(\mu_{\mathrm{I}}, E)\big|_{\mathrm{SCET_I}} = C_i(E, \mu)\, \zeta_M(\mu, E)\big|_{\mathrm{SCET_{II}}}. \tag{50}$$

This relation is not as dull as it seems; rather, it contains the remarkable message that for the soft overlap contribution to heavy-to-light form factors the intermediate hard-collinear scale is without *any* physical significance. Switching from SCET$_{\mathrm{I}}$ to SCET$_{\mathrm{II}}$ we merely describe the same physics using a different set of degrees of freedom. In other words, there is no use of going through an intermediate effective theory. The RG evolution of the soft functions ζ_M remains the same all the way from the high-energy scale $E \sim m_B$ down to hadronic scales $\mu \sim \Lambda$. The physics of the soft overlap term is thus rather different from the physics of the spin-symmetry breaking corrections in the factorization formula (4), for which the hard-collinear scale is of physical significance. Any spin-symmetry breaking contribution involves at least one hard-collinear gluon exchange, and the amplitude factorizes below the scale μ_{hc}.

The result (50) allows us to systematically resum the short-distance logarithms arising in the evolution from high energies down to hadronic scales. The RG equation obeyed by the Wilson coefficient functions is [12, 23]

$$\frac{d}{d\ln\mu} C_i(E, \mu) = \left(\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)] \ln\frac{2E}{\mu} + \gamma[\alpha_s(\mu)] \right) C_i(E, \mu), \tag{51}$$

where the coefficient of the logarithmic term is determined by the cusp anomalous dimension [29]. Its solution is

$$C_i(E, \mu) = C_i(E, \mu_h) \, \exp U(\mu_h, \mu, E), \tag{52}$$

where $\mu_h \sim 2E$ is the high-energy matching scale for the transition from QCD to SCET, at which the values of the Wilson coefficients can be reliably computed using fixed-order perturbation theory. The RG evolution function can be written as

$$U(\mu_h, \mu, E) = \int\limits_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} d\alpha \, \frac{\Gamma_{\mathrm{cusp}}(\alpha)}{\beta(\alpha)} \left[\ln\frac{2E_\gamma}{\mu_h} - \int\limits_{\alpha_s(\mu_h)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')} \right] + \int\limits_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} d\alpha \, \frac{\gamma(\alpha)}{\beta(\alpha)}, \tag{53}$$

where $\beta(\alpha_s) = d\alpha_s/d\ln\mu$. The dependence on the high-energy matching scale μ_h cancels against that of the Wilson coefficients $C_i(E, \mu_h)$ in (52). Note that after exponentiation the evolution function contains an energy and scale-dependent factor $\exp U(\mu_h, \mu, E) \propto E^{a(\mu)}$, where

$$a(\mu) = \int^{\alpha_s(\mu)} d\alpha \, \frac{\Gamma_{\mathrm{cusp}}(\alpha)}{\beta(\alpha)}. \tag{54}$$

In order for this scale dependence to be canceled, the low-energy hadronic matrix element must carry an energy dependence of the form $(\Lambda/E)^{a(\mu)}$.

Evaluating the hadronic matrix elements at a low scale means that all *short-distance* dependence on the large energy E is extracted and resummed in the coefficient functions. However, in the present case the matrix elements still contain a long-distance dependence on the large scale E, which cannot be factorized [23]. The reason is that the large energy enters the effective theory as an external variable imprinted by the particular kinematics of soft-to-collinear

transitions. Because of the large Lorentz boost $\gamma = E/m_M$ connecting the rest frames of the B meson and the light meson M, the low-energy effective theory knows about the large scale E even though hard quantum fluctuations have been integrated out. This is similar to applications of heavy-quark effective theory to $b \to c$ transitions, where the fields depend on the external velocities of the hadrons containing the heavy quarks, and $\gamma = v_b \cdot v_c$ is an external parameter that appears in the matrix elements of velocity-changing current operators [30, 31]. In the present case, it is perhaps reasonable to assume that the primordial energy dependence of the hadronic matrix elements at some low hadronic scale might be moderate, so that the dominant E dependence is of short-distance nature and can be extracted into the Wilson coefficient functions. However, there will always be some energy dependence left in the matrix elements; even if we assume that it is absent for some value of μ, it will unavoidably be reintroduced when we change the scale. As a result, it is impossible to determine the asymptotic behavior of the QCD form factors $f_i^{B \to M}(q^2)$ using short-distance methods.[5]

Let us now proceed to study the numerical importance of short-distance Sudakov logarithms. Given the exact results in (52) and (53), it is straightforward to derive approximate expressions for the resummed Wilson coefficients at a given order in RG-improved perturbation theory by using perturbative expansions of the anomalous dimensions and β-function. Unfortunately, controlling terms of $O(\alpha_s)$ in the evolution function U would require knowledge of the cusp anomalous dimension to three-loop order (and knowledge of γ to two-loop order), which at present is lacking. We can, however, control the dependence on the recoil energy E to $O(\alpha_s)$. Following [24], we define the ratio $r = \alpha_s(\mu)/\alpha_s(\mu_h)$ and obtain

$$
e^{U(\mu_h,\mu,E)} = e^{U_0(\mu_h,\mu)} \left(\frac{2E}{\mu_h} \right)^{-\frac{\Gamma_0}{2\beta_0} \ln r} \left[1 - \frac{\alpha_s(\mu_h)}{4\pi} \frac{\Gamma_0}{2\beta_0} \left(\frac{\Gamma_1}{\Gamma_0} - \frac{\beta_1}{\beta_0} \right) (r-1) \ln \frac{2E}{\mu_h} \right], \qquad (55)
$$

where

$$
U_0(\mu_h,\mu) = \frac{\Gamma_0}{4\beta_0^2} \left[\frac{4\pi}{\alpha_s(\mu_h)} \left(1 - \frac{1}{r} - \ln r \right) + \frac{\beta_1}{2\beta_0} \ln^2 r - \left(\frac{\Gamma_1}{\Gamma_0} - \frac{\beta_1}{\beta_0} \right) (r - 1 - \ln r) \right]
$$
$$
- \frac{\gamma_0}{2\beta_0} \ln r + O(\alpha_s). \qquad (56)
$$

The only piece missing for a complete resummation at next-to-next-to-leading order is the $O(\alpha_s)$ contribution to U_0, which is independent of E. The relevant expansion coefficients are $\Gamma_0 = \frac{16}{3}$, $\Gamma_1 = \frac{1072}{9} - \frac{16}{3}\pi^2 - \frac{160}{27}n_f$, $\gamma_0 = -\frac{20}{3}$, and $\beta_0 = 11 - \frac{2}{3}n_f$, $\beta_1 = 102 - \frac{38}{3}n_f$. We set $n_f = 4$ in our numerical work.

In Figure 6 we show the dependence of the Wilson coefficients $C_i(E,\mu)$ on the large energy E. We choose $\mu_h = 2E$ for the high-energy matching scale and use the tree-level initial conditions $C_i(E,\mu_h) = 1$, which is consistent at next-to-leading order. We fix μ at a low hadronic scale in order to maximize the effect of Sudakov logarithms. The maximum recoil energy in $B \to \pi$ transitions is such that $2E_{\max} \simeq 5.3\,\mathrm{GeV}$. Obviously, for such values the perturbative resummation effects are very moderate. In the energy range $1\,\mathrm{GeV} < 2E < m_B$

---

[5]The same phenomenon is known to occur in the case of the Sudakov form factor, for which the coefficient of the double logarithm is sensitive to infra-red physics [32, 33].
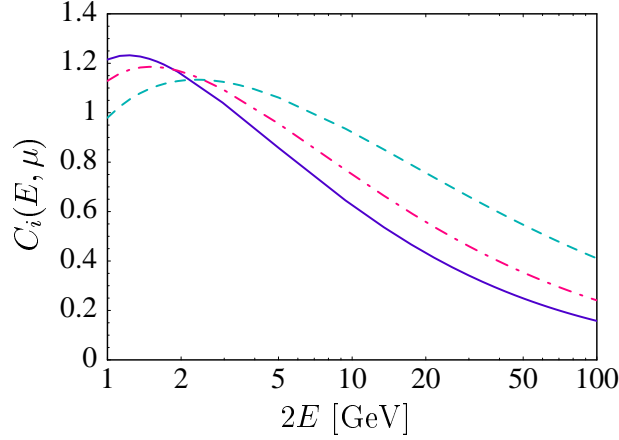
Figure 6: Energy dependence of the Wilson coefficients $C_i(E, \mu)$ at next-to-leading order in RG-improved perturbation theory. The three curves correspond to $\alpha_s(\mu) = 1$ (solid), $\alpha_s(\mu) = 0.75$ (dashed-dotted), and $\alpha_s(\mu) = 0.5$ (dashed), which are representative of typical hadronic scales.

the Wilson coefficients differ from unity by no more than about 20%. The extrapolation to larger energy values shows that Sudakov suppression remains a moderate effect even for very large recoil energy.

7 Factorization of spin-symmetry breaking effects

Using the close connection between messenger exchange and endpoint singularities discussed in Section 4, the formalism of soft-collinear fields can be used to demonstrate the convergence of convolution integrals in QCD factorization theorems. If messenger exchange is unsuppressed, the convolution integrals diverge at the endpoints, spoiling factorization. By the same reasoning, convolution integrals are finite if soft-collinear messenger contributions are absent at leading power.

Let us apply this method to show, to all orders in perturbation theory, that the convolution integrals entering the spin-symmetry breaking term in the factorization formula (4) are free of endpoint singularities. As mentioned in the Introduction, this is an essential ingredient still missing from the proof of this formula. With our technology the proof is rather simple and consists of only the following two steps:

1. Soft-collinear messenger fields can be decoupled from the four-quark operators mediating spin-symmetry breaking effects, which are of the type shown in (30). The reason is that in the color singlet-singlet case the operators are invariant under the field redefinition (13) [23]. It follows that their matrix elements factorize into separate matrix elements of soft and collinear currents, which can be written in terms of the leading-order LCDAs of the external mesons. If the messengers did not decouple, they would introduce unsuppressed interactions between the soft and collinear parts of the four-quark operators, thereby spoiling factorization.

2. Time-ordered products containing interactions of soft-collinear messengers with soft or

collinear fields do not contribute to the spin-symmetry violating term in (4). This follows from SCET power counting. The insertions of the Lagrangians $\mathcal{L}_{\bar{q}\theta}$ and $\mathcal{L}_{\bar{\theta}\xi}$ in (43) cost a factor $\lambda^{3/2}$, meaning that they can only come together with a leading-order current operator of the type $\bar{\mathcal{X}} \ldots \mathcal{H} \sim \lambda^{5/2}$. (Note that there are no $O(\lambda^{1/2})$ corrections to the current that could make the vacuum correlator (44) involving $\mathcal{L}_{\bar{q}\theta}^{(1/2)}$ and $\mathcal{L}_{\bar{\theta}\xi}^{(1/2)}$ non-zero [23].) However, such a current will always be of the form (42) because of the projection properties of the heavy-quark and collinear-quark spinors. It thus respects the spin-symmetry relations.

These two observations imply that at leading order in Λ/E soft-collinear exchanges do not contribute to the spin-symmetry breaking term in the factorization formula, and hence the corresponding convolution integrals are convergent.

8 Summary and conclusions

We have presented a complete scale separation for the soft universal form factors $\zeta_M(E)$, which parameterize the leading spin-symmetric contributions to heavy-to-light form factors at large recoil energy. These functions are related to hadronic matrix elements of operators in soft-collinear effective theory (SCET$_{\text{II}}$). When evaluated using the standard (four-dimensional) projections onto meson light-cone distribution amplitudes, these matrix elements times their respective Wilson coefficient functions lead to singular convolution integrals over light-cone momentum fractions. We have shown that this is an indication that exceptional momentum contributions corresponding to highly asymmetric Fock states of the external mesons cannot be neglected at leading power. In the effective theory, these contributions are parameterized in terms of matrix elements of operators involving soft-collinear messenger modes, which communicate between the soft and collinear sectors of the theory. Using a toy model, we have demonstrated that messenger exchange precisely accounts for the endpoint configurations in the full theory.

The sum of all effective-theory matrix elements is free of spurious endpoint singularities, but it cannot be factorized into convolutions of hard-scattering kernels with light-cone distribution amplitudes. We have argued that the sum of all operators contributing at leading order in power counting is an eigenvector of the anomalous dimension matrix governing operator mixing in the effective theory. The corresponding eigenvalue coincides with the anomalous dimension of the leading-order SCET$_{\text{II}}$ current operator containing a heavy quark and a collinear quark. Using this result, we have performed a complete resummation of short-distance Sudakov logarithms arising in the evolution from the hard scale $\mu \sim 2E \sim m_B$ down to a low hadronic scale $\mu \sim \Lambda$. For the physical value of the B-meson mass, the resummed Sudakov factor leads to no suppression at all. Even for much larger values $m_B \approx 50\text{--}100\,\text{GeV}$, the suppression would only be a factor 2–3. This is insufficient to suppress the soft overlap contribution. The main conclusion of our work is therefore that the soft overlap contribution to heavy-to-light form factors exists, and that it does not receive a significant perturbative suppression.

A perhaps surprising finding of our analysis is that the intermediate hard-collinear scale $\sqrt{E\Lambda}$ is irrelevant to the physics of the soft overlap contribution. This is, at first sight, a puzzling result, because the hard-collinear scale seems to set the natural scale for the gluon

virtualities in Feynman graphs contributing to the form factors, and more generically it is (in many cases) the characteristic scale for the interactions of soft and collinear degrees of freedom. Also, at the hard-collinear scale one usually switches between different effective field theories (called SCET_I and SCET_II) and hence describes physics using different sets of degrees of freedom. Nevertheless, we have shown that the evolution of the soft functions $\zeta_M(E)$ remains the same all the way from the high-energy scale $\mu \sim m_B$ down to hadronic scales $\mu \sim \Lambda$. The physics of the soft overlap term is thus rather different from the physics of the spin-symmetry breaking corrections in the factorization formula (4), for which the hard-collinear scale is of physical significance.

Another unusual (although in retrospect not surprising) result of our study is the observation that the hadronic matrix elements parameterizing the soft overlap contribution contain a long-distance dependence on the large scale E, which cannot be factorized using short-distance methods. The large energy enters the effective theory as an external variable imprinted by the particular kinematics of soft-to-collinear transitions, which are characterized by a large Lorentz boost connecting the rest frames of the B meson and the light meson M. The best that can be achieved is to extract all *short-distance* dependence on the recoil energy by evaluating the matrix elements at a low scale. While it is reasonable to assume that the primordial energy dependence of the hadronic matrix elements at some low hadronic scale might be moderate, it is impossible to determine the precise asymptotic behavior of the QCD form factors $f_i^{B \to M}$ using short-distance methods.

These results have important phenomenological implications. First, the fact that the soft overlap contribution is unsuppressed implies that the spin-symmetric contributions to the form factors are parametrically larger than spin-symmetry violating corrections, which are suppressed by a factor of $\alpha_s(\sqrt{E\Lambda})$. This lends credibility to the idea of an approximate spin symmetry realized in the large-energy limit of QCD. Secondly, our finding that the soft overlap is not significantly suppressed by a short-distance Sudakov factor casts doubt on one of the key assumptions underlying the pQCD approach to exclusive B decays [9]. It supports the power counting scheme employed in the QCD factorization approach, where form factors are treated as non-perturbative hadronic input parameters [5]. Finally, it should be obvious from our discussion that the physics underlying heavy-to-light form factors at large recoil is subtle and far more complicated than the "wave-function overlap" picture usually associated with simpler form factors such as those relevant to $K \to \pi$ or $B \to D$ transitions. We feel that the physics associated with the transformation of the soft degrees of freedom in the B meson into the energetic partons of the light final-state meson is not properly accounted for in existing model calculations of heavy-to-light form factors using quark models or QCD sum rules. A more detailed investigation of the numerical implications of our findings is left for future work.

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Erratum:

Factorization and the soft overlap contribution to heavy-to-light form factors

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Abstract

We correct an argument about the $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$ matching of the soft-overlap contribution to heavy-to-light form factors at large recoil.

In Section 6 entitled "Operator mixing and Sudakov logarithms" of our work [1], we stated incorrectly that only linear combinations of the operators $O_k^{(M)}$ that include $O_5^{(M)}$ can contribute to the $B \to M$ form factors. The argument that led to this conclusion, presented after eq. (47), is erroneous. Linear combinations of the operators $O_k^{(M)}$ with $k = 1, \ldots, 4$ may also contribute, provided that the corresponding Wilson coefficients are free of endpoint singularities. As a result, eq. (48) in [1] is incorrect, and eq. (50) must be generalized to read

$$C_i(E, \mu_I)\, \zeta_M(\mu_I, E)\big|_{\mathrm{SCET_I}} = C_i(E, \mu)\, \zeta_M(\mu, E)\big|_{\mathrm{SCET_{II}}} \tag{50}$$

$$+ \sum_{k=1}^{4} \left[C_i(E, \mu_{hc})\, D_k^{(M)}(E, \mu_{hc}, \mu) - C_i(E, \mu)\, d_k^{(M)}(E, \mu) \right] \otimes \xi_k^{(M)}(\mu)\,,$$

where $\xi_k^{(M)}(\mu)$ denote the factorizable $B \to M$ matrix elements of the operators $O_k^{(M)}$ in eqs. (21) and (22), which can be expressed in terms of twist-2 and twist-3, two- and three-particle light-cone distribution amplitudes of the B meson and the light meson M. The main conclusion of our paper, that there exists an unsuppressed "soft overlap" contribution to the form factors (the first term on the right-hand side in the above equation) for which the hard-collinear scale $\mu_{hc} \sim \sqrt{E\Lambda}$ is irrelevant, remains unchanged. The coefficient functions in the second line of eq. (50), on the other hand, in general do contain logarithms of the hard-collinear scale. An explicit calculation of these coefficients could help to shed light on the question about the numerical size of the soft overlap piece.

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